# MAXIMILIAN SOHMEN

# SUPERSOLIDITY IN DIPOLAR QUANTUM GASES IN AND OUT OF EQUILIBRIUM



# Supersolidity in Dipolar Quantum Gases in and out of Equilibrium

A DISSERTATION PRESENTED BY MAXIMILIAN SOHMEN

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NOBODY IS PERFECT. If you, reader, spot an erroneous or mistakable statement in this work, I would gratefully acknowledge a notification to max.sohmen[at]web.de. Corrected online versions will be released timely.

| rof. Dr. Francesca Ferlaino 🝺, LFU Innsbruck and                        |
|-------------------------------------------------------------------------|
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|                                                                         |



The Flammarion engraving («Un missionnaire du moyen âge ...»). Unknown artist. [97, p. 163]

Nach all den Jahren kann ich immer noch nicht fassen: Woraus hat sich dieser ganze Kram erschaffen? Hat er sich von Null auf Hundert in den Raum gesetzt, Ohne zu fragen, vom Nichts ins Jetzt? [...] Also kehren wir zurück An den Ursprung des Ursprungs des Ursprungs des Ursprungs Zurück an den sogenannten Eisprung des Ursprungs. Und was davor ist, ist dann quasi meine Quelle, An die ich jetzt mal jene kühne Frage stelle: "Warum?"

- Käptn Peng und die Tentakel von Delphi, "Sockosophie"

TO MY TEACHERS, IN PHYSICS AND IN LIFE.

# SYNOPSIS

In experimental physics, quantum gases of ultracold atoms have proved, on the one hand, an extremely productive subject of study for its own sake, and on the other hand, a helpful tool to simulate and understand other, less accessible quantum many-body systems. The vast majority of quantum gas experiments so far has utilised atoms that interact through an isotropic, short-ranged, contact-type interaction. More recently, experiments using atoms of strongly magnetic elements such as chromium, erbium, and dysprosium have been realised, whose physics can be heavily influenced by the anisotropic and longranged dipole-dipole interaction.

One part of this thesis reports on studies of three-dimensional bulk quantum gases of erbium and dysprosium sufficiently confined in the direction of dipole polarisation. In such systems, a carefully balanced interplay between repulsive contact interaction, attractive dipole-dipole interaction, and fluctuations can lead to the spontaneous formation of density-modulated states. Within a narrow parameter regime, these states can combine features of a crystal (spatial periodicity) and of a superfluid (phase coherence), manifesting a so-called supersolid – a counter-intuitive phase of matter whose existence has been speculated about for more than 60 years. A first set of experiments demonstrates the creation of supersolid states from Bose–Einstein condensates of <sup>166</sup>Er as well as <sup>164</sup>Dy through a ramp of the contact interaction, and allows comparison to a phase diagram calculated from an extended Gross-Pitaevskii equation. Dysprosium supersolids are not only found to be particularly long-lived, but also to be producible through direct evaporation from a thermal gas. This process is studied more closely in a second set of experiments. It is observed that during evaporation into the supersolid state the translational symmetry is broken before the phase symmetry, and that thermal excitations enhance the measured degree of density modulation. A third set of experiments is aimed at dysprosium supersolids brought out of equilibrium. It is found that after a contact interaction quench that breaks the supersolid into an incoherent array of isolated quantum droplets, global phase coherence can be restored by increasing the Josephson coupling between the droplets.

Another part of this thesis reports on the development and construction of a quantum gas microscope for dipolar erbium and dysprosium atoms. Single-atom-resolved imaging of ultracold lattice gases has in the past enabled impactful studies of contact-interacting many-body systems. The extension of this technique to dipolar systems will allow to probe extended Bose–Hubbard and extended Fermi–Hubbard models, opening new research perspectives and allowing for advanced quantum simulation scenarios.

# ZUSAMMENFASSUNG

In der Experimentalphysik haben sich Quantengase aus ultrakalten Atomen einerseits selbst als extrem ergiebiges Forschungsobjekt erwiesen, andererseits können sie als hilfreiches Werkzeug dienen, andere, weniger zugängliche Quantenvielteilchensysteme zu simulieren und besser zu verstehen. Die überwältigende Mehrheit von Quantengasexperimenten nutzt nach wie vor Atome, welche miteinander über eine isotrope, kurzreichweitige Kontaktwechselwirkung interagieren. Demgegenüber wurden vor kürzerer Zeit auch Experimente mit stark magnetischen Elementen wie Chrom, Erbium und Dysprosium verwirklicht, deren Physik stark von der anisotropen und langreichweitigen Dipol-Dipol-Wechselwirkung geprägt sein kann.

Ein Teil dieser Dissertation berichtet von Experimenten mit Erbium- und Dysprosium-Quantengasen in dreidimensionalen optischen Fallen, welche deren räumliche Ausdehnung in Dipolrichtung stark eingeschränken. In solchen Systemen kann ein sorgfältig eingestelltes Wechselspiel zwischen abstoßender Kontaktwechselwirkung, anziehender Dipol-Dipol-Wechselwirkung sowie Fluktuationen zur spontanen Ausbildung von dichtemodulierten Vielteilchen-Quantenzuständen führen. Innerhalb eines engen Parameterregimes können solche Zustände gleichzeitig Charakteristika eines Kristalls (räumliche Periodizität) sowie einer Supraflüssigkeit (Phasenkoheränz) aufweisen, und damit die Kriterien für einen sogenannten Suprafestkörper erfüllen – einen kontraintuitiven Materiezustand, über dessen Existenz schon seit mehr als 60 Jahren spekuliert wird. In einer ersten Reihe von Experimenten wird gezeigt, dass Bose-Einstein-Kondensate sowohl aus <sup>166</sup>Er-, als auch aus <sup>164</sup>Dy-Atomen mittels einer Rampe der Kontaktwechselwirkungsstärke in suprafeste Zustände überführt werden können. Die experimentellen Ergebnisse werden mit dem Phasendiagramm verglichen, welches sich aus einer erweiterten Gross-Pitaevskii-Gleichung errechnen lässt. Es zeigt sich außerdem, dass Dysprosium-Suprafestkörper nicht nur besonders langlebig sind, sondern auch durch direkte Evaporation aus einem thermischen Gas hergestellt werden können. Dieser Prozess wird in einer zweiten Reihe von Experimenten eingehender untersucht. Dort wird beobachtet, dass während der Evaporation zum suprafesten Zustand zeitlich die Translationssymmetrie vor der Phasensymmetrie gebrochen wird, und dass thermische Anregungen den gemessenen Grad an Dichtemodulation erhöhen. Eine dritte Reihe an Experimenten beschäftigt sich mit Dysprosium-Suprafestkörpern außerhalb ihres Gleichgewichtszustands. Eine abrupte Veränderung der Kontaktwechselwirkung wird genutzt, um den Suprafestkörper in einzelne, regelmäßig angeordnete, jedoch voneinander isolierte Quantentröpfchen aufzubrechen. Aus diesen kann der suprafeste Zustand wiederhergestellt werden, wenn die Josephson-Kopplung zwischen den Tröpfchen erhöht wird.

Ein anderer Teil dieser Dissertation behandelt die Entwicklung und Konstruktion eines Quantengasmikroskops für dipolare Erbium- und Dysprosiumatome. Die einzelatomaufgelöste Bildgebung von ultrakalten Gasen in optischen Gittern hat in der Vergangenheit bedeutende Studien kontaktwechselwirkender Vielteilchensysteme ermöglicht. Die Ausweitung dieser Technik auf dipolare Systeme erlaubt die Erforschung erweiterter Bose–Hubbard- und Fermi–Hubbard-Modelle, eröffnet damit neue Forschungsperspektiven und Möglichkeiten für fortgeschrittene Quantensimulationsszenarien.

# PUBLICATIONS

Some of the concepts and results presented in this thesis have appeared previously within the following publications:

A. TRAUTMANN,<sup>\*</sup> P. ILZHÖFER,<sup>\*</sup> G. DURASTANTE, C. POLITI, <u>Maximilian Sohmen</u>, M. J. MARK, & F. FERLAINO. "Dipolar Quantum Mixtures of Erbium and Dysprosium Atoms." In: *Physical Review Letters* 121 (2018). DOI: 10.1103/PhysRevLett.121.213601.

L. CHOMAZ, D. PETTER, P. ILZHÖFER, G. NATALE, A. TRAUTMANN, C. POLITI, G. DURASTANTE, Rick M. W. VAN BIJNEN, A. PATSCHEIDER, <u>Maximilian Sohmen</u>, M. J. MARK, & F. FERLAINO. "Long-Lived and Transient Supersolid Behaviors in Dipolar Quantum Gases." In: *Physical Review X* 9 (2019). DOI: 10.1103/PhysRevX.9.021012.

G. DURASTANTE, C. POLITI, <u>Maximilian Sohmen</u>, P. Ilzhöfer, M. J. Mark, M. A. Norcia, & F. Ferlaino. "Feshbach resonances in an erbium-dysprosium dipolar mixture." In: *Physical Review A* 102 (2020). DOI: 10.1103/PhysRevA.102.033330.

P. Ilzhöfer,\* <u>Maximilian Sohmen</u>,\* G. Durastante, C. Politi, A. Trautmann, G. Natale, G. Morpurgo, T. Giamarchi, L. Chomaz, M. J. Mark, & F. Ferlaino. "Phase coherence in out-of-equilibrium supersolid states of ultracold dipolar atoms." In: *Nature Physics* 17 (2021). DOI: 10.1038/s41567-020-01100-3.

Maximilian SOHMEN, C. POLITI, L. KLAUS, L. CHOMAZ, M. J. MARK, M. A. NORCIA, & F. FER-LAINO. "Birth, Life, and Death of a Dipolar Supersolid." In: *Physical Review Letters* 126 (2021). DOI: 10.1103/PhysRevLett.126.233401.

M. A. NORCIA,\* C. POLITI,\* L. KLAUS, E. POLI, <u>Maximilian Sohmen</u>, M. J. MARK, R. N. BISSET, L. SANTOS, & F. FERLAINO. "Two-dimensional supersolidity in a dipolar quantum gas." To appear in: *Nature* (2021). arXiv ID: 2102.05555.

<sup>\*</sup> These authors contributed equally.

Were I the bravest, it were hard, alone, [...] Light is the task, when many share the toil.

— Homer, Iliad, Book 12 [139]

# ACKNOWLEDGEMENTS

It is not that I had expected writing a thesis about quantum gases would be easy. Yet, after more than four years in Innsbruck, I thought I had finally developped a positive and pragmatic attitude towards problems. Oh, how naïve. Writing these lines, I find myself faced with the greatest difficulty of all:

Doing justice to the people who have brought me this far.

Although failure, this time, is inevitable, I will once again draw from the wisdom of Pettis and Stark [229] and try to get things done.

First and foremost, I need to thank Francesca for giving me the opportunity to work in one of her labs. I count myself lucky to have worked in a lab with seemingly unlimited resources, with high aspirations, and for the privilege of having been part of a creative and passionate team that has managed to produce so beautiful results. I thank Francesca for having built this team, and for always motivating me to give my very best.

Second, I owe great thanks to Andreas Läuchli, my co-supervisor, for his advice, for support, and for always being available when I was struggling with questions or difficulties.

Third, I am indebted to Markus Greiner, the third member of my thesis committee. Above all, I thank him for inviting me to spend some time in his group at Harvard, where I have learnt much about quantum gas microscopes and how to build them. Special thanks go to Greg Phelps, Aaron Krahn, Anne Hébert and Sepehr Ebadi, who have warmly welcomed me in the Harvard Erbium team and with whom work has been both fruitful and fun.

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for proud affirmations as well as empirical evidence that the local brew is – at least – up on a par with the Belgian produce.

I fear, at this point, I have already given the unshakable impression that my PhD has been a single long series of happy research trips and holidays, and every counter-argument now pointed out in my defence must only strengthen this impression. In the spirit of Pettis and Stark [229] ("there is no editing stage"), I'll swallow this and just continue.

In the intervals between and after my research trips, I was lucky enough to spend some time<sup>1</sup> in Innsbruck working with brilliant colleagues. Above all, I need to mention my lab seniors. Manfred Mark, a man of quiet expertise, has granted us unwavering support in all questions of technology, and has, as one contribution among many, written the control software in the lab, in a way so proficient and intricate that it took time for us mortals to learn how to wield it.

H. Arno Trautmann, the postdoc during my early time, Heidelberg alumnus and therefore vegetarian<sup>2</sup> has left on me a deep impression of dedication to science, and of a diligent work ethic unbothered by the time of day or night. Further, I have to acknowledge his superiority in karaoke, as well as his continuous strives to disclose to me the infinite benefits of Arduinos and LATEX.

Matt Norcia has been our succeeding postdoc since  $1\frac{1}{2}$  years now, and has injected an awesome JILA 'let's do it' flair of casual pragmatism, intuitive brilliance, and straight goal orientation into the lab. I am infinitely grateful for everything that I have learnt from you, particularly in the broad fields of physics, technology, and diplomacy.

Lauriane Chomaz has formally acted as the postdoc of our group's Erbium Lab, but behind the scenes also contributed greatly to the Er–Dy work, as evidenced by the fact that she is co-authoring all three papers presented in the main body of this thesis. She is, together with Matt, the person I have learnt the most from, and will forever stay an unreachable authority for me in dipolar physics, in climbing, and the science of french cheese.

Now I turn to my motley band of fellow PhD students. First, there is Philipp<sup>3</sup> Ilzhöfer, who has fathered most of the nuts and bolts of today's Er–Dy machine, and made sure each of them is properly tightened. None of the experiments presented in this thesis could have been performed without his devotion to thorough Swabian engineering, fuelled mainly by sugared lemonades and chemical confectionery of diverse kind. May his stern labelling and cable-tying policy forever live on in our lab.

Second, there is Gianmaria Durastante, who was not only the official 'master of tubes and pipes', but has also built a great part of the rest of our setup, while always infusing a healthy level of *dolce vita* into the lab. Here, he has strived to balance the shouting-oriented music of Philipp and H. Arno with some  $\hat{G}\hat{G}^{\dagger}$ , has repeatedly saved me from starvation through his uncanny instincts for spotting free food, and taught me that, worn with his unerring sense of style, every laser goggle can look like a vintage piece from Dolce & Gabbana. Outside the lab, he has been my main companion in ski, bike, and mountain adventures, the majority of them ending up in some sort of river.

<sup>1</sup> According to a quick estimate, around 95.47(2) % of my PhD time. The number in parentheses represents the uncertainty of the last digit, arising from how long it might still take me to finish my acknowledgements.

<sup>2</sup> The vegetarian ratio among the numerous Heidelberg alumni in Innsbruck known to us is 100%.

<sup>3</sup> Please forgive the eternal uncertainty about the distribution of I's and p's in your forename. I hope I got it right.

Third, there is Claudia Politi, who is, despite her level of experience, still a young PhD student. Often has she saved me using her supernatural memory for numbers, by remembering the value of a variable in an analysis script of a measurement set on a certain day where we changed a certain parameter and the weather was bad, or other things the like. I am also grateful for her endless (but ultimately unsuccessful) attempts to introduce me to the universe of dos and do'nts in the Italian cuisine.

Finally, there is Lauritz Klaus, the vegetarian Heidelberg alumnus who joined our lab most recently. As a flatlander from the far north, he has not only lifted our level of the 'real' German language again,<sup>4</sup> but also proved a motivated and skilled apprentice. With him, I deem the experiment and especially some unfinished instrumentation in the preparatory Lab 6<sup>5</sup> in good hands for the future. I also need to acknowledge his lunch loyalty to Bresso<sup>®</sup> from M-Preis, in which he has remained steadfast despite harassment, ridicule, and health advice.

I could go on like this, but in the sake of Pettis and Stark [229], I might have to humble myself and hurry up a bit. I would like to thank all other postdocs – Bing Yang, Tom Bland, and אדרי – as well as PhD students – Simon Baier, Jan-Hendrik Becher, Giulia Faraoni, Dani Petter, Alex Patscheider, Gabriele Natale, Elena Poli and Julián Maloberti – of the Ferlaino group for creating such a friendly, productive, and motivating nest.

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<sup>4</sup> Standards had rapidly declined after the departure of H. Arno.

<sup>5</sup> A forlorn place feared by master and PhD students alike, its porch adorned by some lost soul with a telltale quote from Dante's *Inferno*: "Lasciate ogni speranza, voi ch'entrate!"

<sup>6</sup> Sandra later joined the group of Selim Jochim in Heidelberg, and has, as I have learnt recently, consequently become a vegetarian since.

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# ACRONYMS

| AC                                                                                                           | alternating current                                                                                                                                                                                                                                                                                                                                                                   |
|--------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AISI                                                                                                         | American Iron and Steel Institute                                                                                                                                                                                                                                                                                                                                                     |
| AOD                                                                                                          | acousto-optic deflector                                                                                                                                                                                                                                                                                                                                                               |
| BDG                                                                                                          | Bogoliubov-de Gennes                                                                                                                                                                                                                                                                                                                                                                  |
| BEC                                                                                                          | Bose-Einstein condensate                                                                                                                                                                                                                                                                                                                                                              |
| BKT                                                                                                          | Berezinskii-Kosterlitz-Thouless                                                                                                                                                                                                                                                                                                                                                       |
| CAD                                                                                                          | computer-aided design                                                                                                                                                                                                                                                                                                                                                                 |
| CCD                                                                                                          | charge-coupled device                                                                                                                                                                                                                                                                                                                                                                 |
| CNC                                                                                                          | computer numerical control                                                                                                                                                                                                                                                                                                                                                            |
| CF                                                                                                           | con-flat                                                                                                                                                                                                                                                                                                                                                                              |
| DC                                                                                                           | direct current                                                                                                                                                                                                                                                                                                                                                                        |
| DDI                                                                                                          | dipole-dipole interaction                                                                                                                                                                                                                                                                                                                                                             |
| DMD                                                                                                          | digital micromirror device                                                                                                                                                                                                                                                                                                                                                            |
| DOI                                                                                                          | digital object identifier                                                                                                                                                                                                                                                                                                                                                             |
| EOM                                                                                                          | electro-optic modulator                                                                                                                                                                                                                                                                                                                                                               |
| FEM                                                                                                          | finite-element method                                                                                                                                                                                                                                                                                                                                                                 |
|                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                       |
| FOV                                                                                                          | field of view                                                                                                                                                                                                                                                                                                                                                                         |
| FOV<br>FWHM                                                                                                  | field of view<br>full width at half maximum                                                                                                                                                                                                                                                                                                                                           |
| FOV<br>FWHM<br>GPE                                                                                           | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation                                                                                                                                                                                                                                                                                                              |
| FOV<br>FWHM<br>GPE<br>IC                                                                                     | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation                                                                                                                                                                                                                                                                                      |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY                                                                              | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang                                                                                                                                                                                                                                                                    |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA                                                                       | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation                                                                                                                                                                                                                                     |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO                                                                | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order                                                                                                                                                                                                                 |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO<br>MOPA                                                        | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier                                                                                                                                                                            |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT                                                 | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap                                                                                                                                                    |
| FOV<br>FWHM<br>GPE<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA                                                 | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture                                                                                                                              |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA<br>NA                                     | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture<br>non-evaporable getter                                                                                                     |
| FOV<br>FWHM<br>GPE<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA<br>NA<br>NEG<br>OD                              | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture<br>non-evaporable getter<br>optical depth                                                                                    |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA<br>NEG<br>OD                              | field of view<br>full width at half maximum<br>Gross-Pitaevskii equation<br>infinite conjugation<br>Lee-Huang-Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture<br>non-evaporable getter<br>optical depth                                                                                    |
| FOV<br>FWHM<br>GPE<br>IC<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA<br>NA<br>NA<br>NEG<br>OD<br>CODLRC<br>ODT | field of view<br>full width at half maximum<br>Gross-Pitaevskii equation<br>infinite conjugation<br>Lee-Huang-Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture<br>non-evaporable getter<br>optical depth<br>off-diagonal long-range order<br>optical dipole trap                            |
| FOV<br>FWHM<br>GPE<br>LHY<br>LDA<br>LRO<br>MOPA<br>MOT<br>NA<br>NA<br>NEG<br>OD<br>CDLRC<br>ODT<br>ODD       | field of view<br>full width at half maximum<br>Gross–Pitaevskii equation<br>infinite conjugation<br>Lee–Huang–Yang<br>local density approximation<br>long-range order<br>master-oscillator power amplifier<br>magneto-optical trap<br>numerical aperture<br>non-evaporable getter<br>optical depth<br>off-diagonal long-range order<br>optical dipole trap<br>optical path difference |

# xxii acronyms

- PCA principal-component analysis
- PSF point-spread function
- QF quantum fluctuations
- QFT quantum field theory
- RF radio frequency
- RMS root mean square
- RWA rotating-wave approximation
- SCMOS scientific complementary metal-oxide semiconductor
- SIL solid-immersion lens
- SNR signal-to-noise ratio
- SNOM scanning near-field optical microscopy
- TDL thermodynamic limit
- TOF time of flight
- UHV ultra-high vacuum
- UV ultraviolet
- USAF United States Air Force
- WD working distance

The ability to reduce everything to simple fundamental laws does not imply the ability to start from these laws and reconstruct the universe.

- Philip W. Anderson, "More is different" [6]

# WHY STUDY QUANTUM GASES?

The experimental realisations of Bose–Einstein condensates (BECs) [5, 45, 47, 74] and degenerate Fermi gases [76, 198, 261, 285] more than twenty years ago marked important scientific breakthroughs, which can be seen both as a finish line, the culmination of decades of efforts in cooling and trapping of uncharged atoms, as well as the starting gun for an entirely new research direction.

On the one hand, experiments on ultracold atoms are exciting in their own right, and continue to deliver deep and sometimes surprising insights, as for example into the decade-old debate about supersolidity.

On the other hand, ensembles of ultracold atoms can serve as universal model systems for other quantum many-body systems since they are (i) experimentally accessible, offer (ii) a high degree of controllability of system quantities over (iii) an extremely large parameter range, as well as (iv) efficient probing techniques. In these respects, for example, studying atoms in an optical lattice has many advantages over studying electrons in a metal [34, 35, 175]. Often, such *quantum simulator* approaches are also required from the theory side since, as pointed out by Feynman [95], the computational power needed to describe an ensemble of quantum particles scales exponentially with the number of constituents, setting practical limits for numeric calculations. The current state of the art in technology for quantum simulation using atoms in optical lattices is set by so-called quantum gas microscopes, which allow to image single atoms on individual lattice sites (cf., e.g., Refs [34, 122], and references therein).

#### WHAT IS KNOWN ABOUT DIPOLAR QUANTUM GASES, WHAT NOT?

At the start of my work in the Er–Dy laboratory in the Ferlaino group, degenerate Bose and Fermi gases of strongly magnetic elements had already been produced by our group in Innsbruck, using erbium [2, 3], and by the Lev group at Stanford, using dysprosium [52, 184].<sup>1</sup> Similar experiments of other groups, for example the dysprosium laboratories lead by Tilman Pfau in Stuttgart or Giovanni Modugno in Pisa, had gone into operation a bit later. None of these first-generation laboratories, however, has the capability to study heteronuclear mixtures of two different dipolar elements.

In the field of bulk dipolar quantum gases, in 2016 a publication by the Stuttgart group [145] attracted much attention for the surprising observation that after an interaction quench to a regime where, according to traditional mean-field theory, the collapse of a

<sup>1</sup> BECs of the comparatively weakly magnetic element chromium had already been obtained some time earlier in Stuttgart [116] and Paris [22].

dysprosium BEC had been expected, instead arrays of stable, yet highly-excited and phaseuncorrelated quantum droplets were found. Subsequent works, from both theory [28, 288] and experiments by the Stuttgart [91, 259] and by our group [65, 66] helped associate the underlying stabilisation mechanism to a beyond-mean-field effect. The crucial question remained, however, whether the roton mode population leading to the formation of the phase-incoherent dipolar droplet arrays in these early experiments was a mere result of the high degree of excitation, introduced through the interaction quench as well as finite temperature, or if, in principle, droplet arrays could also constitute a novel type of system ground state. If so, it was immediately realised that an array of phase-locked droplets could open an experimental path towards creating a supersolid state of matter. The nature of such a – back then still hypothetical – dipolar supersolid was a subject of much speculation, for example concerning its spectrum of excitations, its reaction to external perturbations or a finite-temperature thermal bath, as well as the order of the transitions between phases of different symmetry.

In the field of atoms in optical lattices, a major breakthrough has been the realisation of the first quantum gas microscopes at Harvard and Munich in 2009/10 [18, 267], demonstrating single-atom resolution for the bosonic isotope <sup>87</sup>Rb. Microscopes for further, non-dipolar bosonic and fermionic species have been demonstrated by other laboratories some time later [60, 84, 129, 197, 211, 214, 297].

Specifically for dipolar atoms, extended Bose–Hubbard models [13] and strongly interacting fermions in optical lattices [16] have been studied in our group using erbium atoms, albeit without microscope resolution. A quantum gas microscope offering single-atom resolution for strongly dipolar species such as erbium or dysprosium does not yet exist.

# OUTLINE OF THIS THESIS, AND A PEEK AT THE MAIN RESULTS

This thesis is a whole divided into three parts. The first part, §§ 1–4, provides an introduction into the theory of dipolar quantum gases. This includes (§ 1) a recapitulation of quantum statistics and one of its direct implications, Bose–Einstein condensation, (§ 2) particle interactions of contact and dipolar type, (§ 3) the influence of particle interactions on the theory of Bose–Einstein condensates on mean-field and beyond-mean-field level, as well as (§ 4) an introduction to basic notions underpinning the concept of supersolidity.

The second part (§§ 5–9) is devoted to experiments with dipolar quantum gases in the bulk. In § 5 the Innsbruck Er–Dy experiment is described, the first and – so far – only apparatus in the world capable of producing heteronuclear quantum mixtures of two magnetic elements [82, 283]. A strong emphasis in the experiment description is put on a newly developped and implemented high-resolution imaging system which allows to probe quantum gases (and mixtures) of erbium and dysprosium directly in trap. In § 6, the twisted path of historic key events leading up to the experimental realisation of dipolar supersolids is sketched.

In §7, our publication on observation transient and long-lived supersolid behaviour in quantum gases of erbium and dysprosium is presented [67]. In this publication, we report on the creation of dipolar quantum gases with supersolid properties and find a remarkable degree of agreement between, on the one hand, experimental data samples of both erbium and dysprosium, prepared via an interaction-ramp sequence, and, on the other hand, zero-temperature phase diagrams spanning the BEC–supersolid phase transition, computed from an extended Gross–Pitaevskii theory. Furthermore, we report the first production of a quantum gas with supersolid properties not involving an interaction quench, but through direct evaporation from a thermal gas.

In §8, our publication on an in-depth study of the phase transition from a thermal gas into a dipolar supersolid state is covered [272]. In this work, we find that during the evaporation process, the translational symmetry is broken before the phase symmetry, and we discuss the impact of temperature on the formation of density-modulated states.

Last, in § 9, our publication on phase coherence in dipolar supersolids out of equilibrium is presented [143]. In this work, we investigate experimentally how the global coherence in a dipolar supersolid state gets lost if, by a quench of contact interactions, collective excitations are introduced and the density links between droplets are depleted. We find that in the resulting incoherent state, global phase coherence can be re-established if the droplet density links are restored by a second ramp of contact interactions. The essential behaviour is found to agree well with a Josephson junction array model; effects that hint at physics beyond the capabilities of this simple model are also elaborated on.

The third part (§§ 11–13) covers the development and engineering of a quantum gas microscope for dipolar atoms. This includes the discussion of initial design variants, a presentation of the development and test of the microscope optics, a description of the complete, newly engineered ultrahigh-vacuum system, the carefully tailored magnetic field coil system as well as a custom multilayer passive magnetic shielding for protection from external influences.

In the appendix, some important concepts and formalisms of (§ A) field theory for quantum gases and (§ B) light propagation and imaging are presented for reference. Furthermore, (§ C) additional, hitherto published works which I have co-authored during my doctoral studies in Innsbruck are presented.

Innsbruck, June 2021 M. S.

Throughout this thesis, if not stated otherwise, for numerical values a number in parentheses (if present) indicates the statistical  $1\sigma$  confidence interval of the last digit(s) of a calculated or measured quantity.

Part I

# BACKGROUND THEORY ON DIPOLAR QUANTUM GASES

A man cannot be too careful in the choice of his enemies.

– Oscar Wilde [296, Ch. 1]

# "QUANTUM" + "GAS"

This chapter serves to introduce the notion of an ideal quantum gas, to motivate why ensembles of quantum particles must follow non-classical statistics, and to sketch how quantum statistics for bosonic particles directly imply a condensation phenomenon at high phase-space density.

## 1.1 TAUTOLOGIES

What is a quantum gas? Even though the name is short, it already entails many implications. In the following, let us therefore try to dissect it a little.

A quantum gas is a gas. This means, specifically:

- 1. The system contains many particles.
- 2. The sample is dilute.

"Many" in Point 1, for example in the context of cold atom experiments, can mean particle numbers *N* often on the order of millions. Each out of these *N* particles has a position, a momentum, and possibly other degrees of freedom such as angular momentum, wherefore the number of degrees of freedom of the entire system is a multiple of *N*. To distil from such a large number of degrees of freedom a few tractable and meaningful observables, statistical tools are needed.

Point 2 characterises a gaseous state insofar that typically the range of interparticle interactions is much smaller than all other relevant length scales of the system, especially than the mean interparticle distance. If this is true, interactions are dominantly two-body.

*A quantum gas is quantum*. This means our system is governed by rules arising from three insights:

- 1. Some measurable properties may only take distinct values.
- 2. Matter has a wave nature.
- 3. Elementary particles are indistinguishable.

Point 1 can be traced back to Max Planck [235, 236], who realised that when we measure certain properties like energy, momentum (and others) within a bound system, the outcomes will not be continuously distributed, but discrete.

Point 2 was advanced by Luis de Broglie [75], demanding that a wave behaviour needs to be attributed to all matter. Consequently, measurement outcomes for quantum particles

are described probabilistically by the square of a wavefunction. A direct implication of constructing particles from a wave basis is Heisenberg's uncertainty relation: some properties of a particle, like position and momentum, are not sharply defined simultaneously. In an ensemble of temperature T one must therefore ascribe to particles of mass m a spatial extent given by the thermal de-Broglie wavelength

$$\Lambda = \hbar \sqrt{2\pi/mk_{\rm B}T},\tag{1}$$

where  $\hbar$  is the reduced Planck constant and  $k_{\rm B}$  is the Boltzmann constant.

Finally, Point 3 must be attributed to Satyendra Nath Bose [43]. Intrinsic properties of elementary particles, such as rest mass and electric charge, are identical. The uncertainty relation prevents simply following particle trajectories at arbitrary precision. Therefore, given two particles with identical intrinsic properties (and other quantum numbers, such as angular momentum) and *sufficiently close* ( $\sim \Lambda$ ) to each other, it is undecidable which is which. Bose realised that indistinguishability implies non-classical particle statistics.

#### 1.2 STATES, SYMMETRY, AND STATISTICS

A single quantum particle is denoted by a Dirac ket which contains the full set of eigenstates of the particle. An *N*-body system is then described by the tensor product of the single-particle kets – or, if we pick a certain representation, a wavefunction (see §§ A.1– A.2). If the particles are indistinguishable, by definition no measurement of a physical quantity can reveal whether two particles have been exchanged or not. This exchange degeneracy implies that the wavefunction has only two possibilities upon particle swapping, to change or not to change sign (see § A.2).

Both of these possibilities are realised in nature. Wavefunctions antisymmetric upon particle exchange belong to fermions, for which the Pauli exclusion principle forbids more than one particle in the same single-particle state. Wavefunctions symmetric upon particle exchange belong to bosons. Compared to classical particles, for indistinguishable bosons the statistical weight of permutations is lost. Therefore, indistinguishable bosons tend to accumulate in the same single-particle state (see Fig. 1).

When investigating systems of large numbers of particles, we have to rely on tools from probability theory. In particular, if we want to calculate the macroscopic thermodynamic properties of a large system from the statistics of the different microscopic particle arrangements, we need the appropriate partition function.

In most experiments, the system under study is not completely isolated but can exchange energy as well as particles with the environment. The appropriate framework for this situation is the grand canonical ensemble, where the chemical potential  $\mu$  controls the variation of particle number as a Lagrange multiplier with respect to the energy *E*. If we choose the energy eigenstates of the non-interacting Hamiltonian with corresponding eigenvalues  $\varepsilon_i$  as basis, the grand partition function (cf. § A.4.1) directly gives the average occupation number of state *i*,

$$\bar{n}_i = \frac{1}{\mathrm{e}^{(\varepsilon_i - \mu)/k_\mathrm{B}T} \pm 1},\tag{2}$$



FIGURE 1: Distribution probabilities for two particles into two states, "left" and "right". The two classical states "white particle left, black particle right" and vice versa (a) merge into a single state "one particle left, one particle right" for indistinguishable (grey) particles. For bosons (b) therefore the probability of finding two particles at the same site is enhanced from  $\frac{1}{2} \rightarrow \frac{2}{3}$ . For identical fermions (c), the probability for double occupancy vanishes due to the Pauli exclusion principle,  $\frac{1}{2} \rightarrow 0$ . Cf. Ref. [96, 106].

where the '+' is for fermionic and the '-' is for bosonic particles. The total number of particles in the system and the total energy are, accordingly,

$$N = \sum_{i} \bar{n}_{i} \quad \text{and} \quad E = \sum_{i} \varepsilon_{i} \bar{n}_{i}. \tag{3}$$

For fermions, Eq. 2 is valid without restrictions. For bosons, in contrast,  $\mu$  has to be lower than the lowest energy level  $\varepsilon_0$  to avoid unphysical negative occupation numbers. If we define our origin of energy as  $\varepsilon_0 \equiv 0$  without loss of generality, we see that for  $\mu \to 0$  the ground state occupation diverges. The fact that this can only happen to the ground state suggests to separate the total number of particles into ground and excited states,

$$N = N_0 + N_T$$
, where  $N_0 = \frac{1}{e^{-\mu/k_B T} - 1}$  and  $N_T = \sum_{i>0} \bar{n}_i$ . (4)

As Einstein already pointed to in 1925 [85], under certain circumstances the number  $N_T$  of available thermal states is limited (see § A.4.3 sq.). All additional particles then must go into  $N_0$ . This can lead to a macroscopic occupation of the ground state, called Bose–Einstein condensation.

Importantly, Bose–Einstein condensation is a purely statistical effect that does not require any particle interactions. It can occur at any temperature, if the density is high enough, or reversely, at any density, if the temperature is low enough. In particular, condensation typically sets in at a critical temperature much higher than the energy gap to the first excited state,  $k_B T_c \gg \varepsilon_1$ . This is in stark contrast to classical Maxwell-Boltzmann statistics, where a macroscopic ground-state occupation is only expected for  $T \ll \varepsilon_1$ . BEC of an ideal gas

1.3 FIELD OPERATORS

In practice, for a many-body system with a large number *N* of indistinguishable particles it can become a difficult task to decide whether the (anti-)symmetry of a state, e.g. in position space,  $|\vec{r}_1, \vec{r}_2, ..., \vec{r}_N\rangle$ , is (i) correct and (ii) conserved when it is evolved into another,  $|\vec{r}'_1, \vec{r}'_2, ..., \vec{r}'_N\rangle$ .

Field theory approaches this problem by introducing field operators  $\hat{\psi}(\vec{r})$  and  $\hat{\psi}^{\dagger}(\vec{r})$  which, respectively, annihilate and create particles at a certain position  $\vec{r}$  and enforce conservation of the Bose (or Fermi) statistics naturally by the way they are constructed. This means we can, on the one hand, generate any arbitrary but properly (anti-)symmetrised many-body state from the vacuum  $|0\rangle \equiv |0_1, 0_2, 0_3, \ldots\rangle$  by repeated application of the creation operator, and on the other hand transfer every many-body state into another using combinations of annihilation and creation operators,

$$|\vec{r}'_1, \vec{r}'_2, \dots, \vec{r}'_N \rangle = \hat{\psi}^{\dagger}(\vec{r}'_1) \hat{\psi}(\vec{r}_1) \, \hat{\psi}^{\dagger}(\vec{r}'_2) \hat{\psi}(\vec{r}_2) \cdots \hat{\psi}^{\dagger}(\vec{r}'_N) \hat{\psi}(\vec{r}_N) \, |\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N \rangle \,.$$

The compliance with the (anti-)symmetry of the system is then naturally ensured through the algebra defined through the (anti-)commutation relations of the field operators (see  $\S$  A.2).

Moreover, it can be shown that with the aid of field operators, any operator in an *N*-body Hilbert space can be extended into Grand Hilbert space, where *N* is allowed to vary (e. g., Ref. [37, 290]). Thus, in a sense, many problems of transforming complicated many-body states simply reduce to determining how the field operator transforms. In particular, since all changes to a given state can be formulated by multiplying field operators to it, the time evolution of a state is fully captured by the time-dependent field operator in the Heisenberg picture, as can be seen straight-forward for a *N*-body ground state  $|G_N\rangle$ :

$$\hat{\psi}_{\rm H}(\vec{r},t) |G_N\rangle = \hat{U}^{\dagger}(t)\hat{\psi}(\vec{r})\hat{U}(t) |G_N\rangle = e^{-iE_0(N)t/\hbar}\hat{U}^{\dagger}(t) |G_{N-1}\rangle$$

$$= e^{-iE_0(N)t/\hbar}e^{iE_0(N-1)t/\hbar} |G_{N-1}\rangle = e^{-i\mu t/\hbar}\hat{\psi}(\vec{r}) |G_N\rangle$$
(5)

Here we have used (i) the usual time evolution operator  $U(t) = e^{-i\hat{H}t/\hbar}$ , (ii) the fact that the *N*- and the (*N*-1)-body ground states are eigenstates of the Grand-Hilbert space Hamiltonian<sup>1</sup>,  $\hat{H} |G_N\rangle = E_0(N) |G_N\rangle$ , and (iii) that the definition of the chemical potential,

$$\mu = E_0(N) - E_0(N-1) \approx \frac{\partial E_0}{\partial N},\tag{6}$$

where the approximation on the right-hand side becomes valid for large enough particle numbers.

A convenient way to circumvent the need for explicit (anti-)symmetrisation of states completely is switching to the Fock representation. In Fock representation,  $|n_1, n_2, ...\rangle$ , only the occupation numbers  $n_s$  of the single-particle states  $|s\rangle$  are listed (see § A.2); the unphysical information ("which particle is which?") then does not even show up.

Fock representation

<sup>&</sup>lt;sup>1</sup> Cf. the appendix, § A.2. Since in the following we will always be dealing with states whose particle number is allowed to vary, we will omit marking the Grand-Hilbert space Hamiltonian explicitly,  $\hat{H} \equiv \hat{H}^{G}$ , unless otherwise noted. NB that  $\hat{H}^{g}$  with a lowercase superscript is used to denote the grand-canonical Hamiltonian, in contrast.

It is often advantageous to pick the specific Fock representation where the occupation numbers refer to the energy eigenstates of the system,  $\hat{H} |n_s\rangle = \varepsilon_s \hat{n}_s |n_s\rangle = \varepsilon_s n_s |n_s\rangle$ . The corresponding bosonic single-particle raising and lowering operators are  $\hat{a}_s^{\dagger}$  and  $\hat{a}_s$ , and the number operator is  $\hat{n}_s = \hat{a}_s^{\dagger} \hat{a}_s$ . In this representation, the field operator takes the form  $\hat{\psi}(\vec{r}) = \sum_s \varphi_s(\vec{r}) \hat{a}_s$ , where  $\varphi_s(\vec{r})$  is the single-particle wavefunction associated with state *s*.

- John S. Bercow, MP

#### 1.4 ORDER, AND HOW TO PARAMETRISE IT

As we have already touched upon in § 1.2, systems of ideal bosonic particles can exhibit a macroscopic occupation  $N_0 \leq N$  of the ground state, whereas the occupation of excited states  $N_{s\neq0}$  is always lesser than  $N_0$  and typically far below unity. This suggests to separate the field operators into the single-particle ground and excited states,

$$\hat{\psi}(\vec{r}) = \varphi_0(\vec{r})\hat{a}_0 + \sum_{s>0} \varphi_s(\vec{r})\hat{a}_s.$$
(7)

Following what is known as the Bogoliubov prescription, we can calculate the expectation value<sup>2</sup> of the annihilation operator with respect to a bosonic number state (with  $N_0 \equiv n_0 \gg 1$ ),

$$\langle \hat{\psi}(\vec{r}) \rangle = \langle N_0, n_1, \dots | \hat{\psi}(\vec{r}) | N_0, n_1, \dots \rangle$$

$$= \sqrt{N_0} \varphi_0(\vec{r}) \underbrace{\langle N_0, n_1, \dots | N_0 - 1, n_1, \dots \rangle}_{\text{TDL} \to 1} + \sum_{s \ge 0} \sqrt{n_s} \varphi_s(\vec{r}) \underbrace{\langle N_0, \dots, n_s, \dots | N_0, \dots, n_s - 1, \dots \rangle}_{(9)$$

**~**1

$$\approx \sqrt{N_0}\varphi_0(\vec{r}).\tag{10}$$

In steps 9-10, we have made the Bogoliubov approximation. The reasoning is that for  $N_0 \gg 1$  the physical properties of the number state  $|N_0, n_1, ...\rangle$  should change little (on the order of  $1/N_0$ ) if a particle is added to or removed from  $N_0$ .<sup>3</sup> Formally, this is only true in the thermodynamic limit (TDL),<sup>4</sup> but in practice already gives sensible results for  $N_0$  on the order of tens of particles.

The Bogoliubov approximation (10) is equivalent to interpreting the ground-state ladder operator as a c-number,  $\hat{a}_0 \rightarrow \sqrt{N_0}$ . We can, in this spirit, rewrite Eq. 7 as

$$\hat{\psi}(\vec{r}) = \sqrt{N_0} \varphi_0(\vec{r}) + \sum_{s>0} \varphi_s(\vec{r}) \hat{a}_s \equiv \Psi(\vec{r}) + \hat{\delta}(\vec{r})$$
(11)

<sup>2</sup> Mathematically, this procedure is somewhat imprecise and controversial (e.g., Ref. [172]), but the implications we are aiming at are sound. In the spirit of Feynman ("it is the facts that matter, not the proofs"), one has to be pragmatic sometimes.

<sup>3</sup> Formally, the approximation of non-orthogonality of number states as above is equivalent to the Hartree-Fock ansatz of a product state [20, 106, 223].

<sup>4</sup>  $N \to \infty$  at N/V = const. and  $N_0/N = \text{const.}$ 

# 8 "quantum" + "gas"

We shall first concentrate on  $\Psi(\vec{r})$ , but later pay special attention to the sum  $\hat{\delta}(\vec{r})$ , known as the fluctuations operator<sup>5</sup>.

Order parameter

Since the Bogoliubov approximation (10) is only valid if the ground-state is macroscopically occupied ( $N_0 \leq N$ ), the complex field  $\Psi(\vec{r})$  acts as an order parameter for the phase transition to a BEC. It may not have been obvious from the beginning that a classical field such as  $\Psi(\vec{r})$  is sufficient to describe the collective properties of a many-body system while properly accounting for the quantum correlations between particles [290].<sup>6</sup> This presents a welcome simplification for computations; however, it is important to note that the order parameter is *not* an ordinary wavefunction. In particular, the time-evolution of ordinary wavefunctions is governed by the energy, whereas the order parameter inherits its timeevolution directly from the field operator (Eq. 5),

$$\Psi_{\rm H}(\vec{r},t) = \hat{U}^{\dagger}(t)\Psi(\vec{r})\hat{U}(t) = e^{-i\mu t/\hbar}\Psi(\vec{r}), \tag{12}$$

hence is governed by the chemical potential  $\mu$ .

Reducing the treatment of the many-body quantum state to the order parameter and neglecting the quantum fluctuations presents a zero-order approximation that works well enough for describing many properties of of ultracold Bose gases. Sometimes, however, certain phenomena cannot be explained without including the effect of quantum fluctuations, as will be detailed in § 3.

<sup>5</sup> Note that  $\hat{\delta}(\vec{r})$  may contain both *quantum* and *thermal* fluctuations, cf. Refs [32, 201]

<sup>6</sup> In this respect, the order parameter can be seen as an analogue to the classical Maxwell description of electromagnetic radiation, which becomes valid when the photon occupation numbers are high [234].

It would be a poor thing to be an atom in a universe without physicists. And physicists are made of atoms. A physicist is the atom's way of knowing about atoms.

- George Wald, "Life and mind in the universe" (1980s)

# PARTICLE INTERACTIONS

While the ideal gas model suffices to explain statistical effects such as Bose–Einstein condensation, other effects lie beyond its capabilities. An important example is the quantum effect of superfluidity, which will be inspected more closely later in this thesis (§§ 4.1 sq.). For non-interacting bosons, the Landau critical velocity (Eq. 67) is zero; without some sort of attractive interaction, fermions cannot even pair up and condense [86]. We will now inspect this missing ingredient: particle interactions.

#### 2.1 CHARACTERISING INTERACTIONS

A vital of assumption in the context of quantum gases is diluteness, meaning that the range of interatomic forces  $r_0$  is much smaller than the mean interparticle distance,  $r_0 \ll n^{-1/d}$ , where *n* is the number density of particles and *d* the dimensionality of the system. Diluteness allows (i) to consider only pairwise interactions and safely neglect three- or more-body processes and (ii) to always consider the asymptotic expression for the scattering amplitude (Eq. 15 below) [234].

An interaction is called short-range if the associated energy is extensive in the thermodynamic limit, i.e. when the energy depends only on the particle density n, whereas it is called long-range if the energy is intensive, i.e. if the energy depends not only on nbut also on the total particle number N. An extensive (intensive) energy is obtained if the integral of the interaction potential,

$$\int_{R}^{\infty} U(\vec{r}) \mathrm{d}^{d}r,\tag{13}$$

evaluated outside a short-distance cut-off *R*, converges (diverges) at long distance *r* (see Ref. [163] and references therein). For central potentials falling off as  $U(\vec{r}) \propto r^{-n}$  this implies

$$d \begin{cases} < \\ \ge \end{cases} n \iff \text{ interaction } \begin{cases} \text{short} \\ \log \end{cases} \text{ range.}$$

$$(14)$$

Generally, a scattering process between two particles is described by a two-body wavefunction that solves the Schrödinger equation. Any elastic scattering process between two particles, however, is equivalent to the scattering of one particle with reduced mass M in a central potential  $U(\vec{r})$ . We therefore consider the centre-of-mass frame and a plane wave representing a particle moving freely along z with energy  $\hbar^2 k^2 / 2M$ . Dropping normalisa2

Interaction range

tions, the full wavefunction at position  $\vec{r}$  may in the far field ( $r \gg r_0$ ) be written as the sum of the incoming plane and an outgoing spherical wave [167, § 123],

$$e^{ikz} + f(k,\vartheta) \frac{e^{ikr}}{r},$$
(15)

Scattering amplitude where  $\vartheta$  is the angle between  $\vec{r}$  and the *z*-axis. The far-field scattering amplitude  $f(k, \vartheta)$  can be expanded in partial waves of angular momentum  $\ell$ ,

$$f(k,\vartheta) = \sum_{\ell=0}^{\infty} (2\ell+1) f_{\ell} P_{\ell}(\cos\vartheta) \quad \text{with} \quad f_{\ell} = \frac{1}{2ik} (e^{2i\delta_{\ell}} - 1), \tag{16}$$

where  $P_{\ell}(\cos \vartheta)$  are the Legendre polynomials and  $\delta_{\ell}$  is a phase shift [ibid.]. The corresponding scattering cross section between identical particles, integrated over the full solid angle, is then

$$\sigma(k) = \int |f(k,\vartheta) \pm f(k,\pi-\vartheta)| d\Omega$$

$$= \sum_{\substack{\ell \text{ even} \\ \text{odd}}} \sigma_{\ell}(k) \quad \text{with} \quad \sigma_{\ell}(k) = 4\pi (2\ell+1)|f_{\ell}|^{2},$$
(17)

where due to symmetry reasons, for bosons (fermions) we have to consider the '+' ('-') sign and the sum runs over even (odd)  $\ell$ . For  $U(\vec{r}) \propto r^{-n}$  as above and low energies  $(k \rightarrow 0)$ , the phase shifts  $\delta_{\ell}$  scale as [167, 254]

$$\delta_{\ell} \propto \begin{cases} k^{2\ell+1} & \text{for } \ell < \frac{1}{2}(n-3), \\ k^{n-2} & \text{otherwise.} \end{cases}$$
(18)

Scattering length

In typical quantum gas experiments the length scales of interest are much greater than the range of interparticle forces ( $r_0$ ), such that the microscopic details of the inter-atomic potential  $U(\vec{r})$  are irrelevant. Instead, the interaction can be fully described by a single parameter, the scattering length a, which is closely connected to the phase shifts  $\delta_{\ell}(k)$  in the low-k limit. Such an approach is called an effective field theory [33].

If the scattering length is small compared to the interparticle spacing,  $|a| \ll n^{-1/d}$ , the gas is weakly interacting. The opposite case,  $|a| \gg n^{-1/d}$ , marks the strongly-interacting or unitary regime [73].

#### 2.2 THE CONTACT INTERACTION

In many cases, the atomic interaction potential is of van-der-Waals type, scaling at long distance as  $U_c(\vec{r}) \propto -C_6/r^6$  with a normalising coefficient  $C_6$ . Such an interaction is always short-ranged by the definition (14), irrespective of dimensionality.

We further see that at low energy  $(k \to 0)$  the cross section (17) is dominated by the term  $\sigma_0(k)$  since  $\sigma_{\ell>0}(k) \to 0$ . In this s-wave regime, the scattering is completely isotropic

Scattering cross section

Interaction strength
and it is legitimate to approximate the detailed, microscopic scattering potential by a delta pseudo-potential<sup>1</sup>,

$$U_{\rm c}(\vec{r}) \to g\delta(\vec{r})$$
 (19)

with an interaction parameter g. It follows for the far-field scattering amplitude and the total cross section

$$f(k)|_{(\ell=0)} = -\frac{a_{\rm s}}{1+{\rm i}ka_{\rm s}}$$
 and  $\sigma(k)|_{(\ell=0)} = \frac{8\pi a_{\rm s}^2}{1+k^2 a_{\rm s}^2}$ , (20)

respectively, with the s-wave scattering length

$$a_{\rm s} = -\lim_{k \to 0} \frac{1}{k} \tan \delta_0(k) = \frac{gM}{2\pi\hbar^2}$$
(21)

and M = m/2 for identical particles of mass m [73].

In an experiment, the scattering length can be set to (almost) arbitrary values if a suitable Feshbach resonance [88, 93] is at hand.<sup>2</sup> Erbium and dysprosium, for example, feature extremely rich Feshbach spectra with a wealth of resonances [82, 103, 189]. Setting a value for the scattering length then only requires to tune the magnitude of a magnetic bias field.

The s-wave scattering rate (20) typically differs greatly between systems of cold bosonic and of cold (identical) fermionic particles. For bosons, it determines, e.g., the rates of thermalisation or three-body loss in a cloud of atoms, and therefore constitutes an important experimental parameter. For fermions, in contrast, only odd  $\ell$  are allowed, such that  $\sigma(k) \rightarrow 0$  in the limit  $k \rightarrow 0$ . Therefore, at low *T*, they will effectively not collide anymore, such that there is neither thermalisation, nor three-body loss. However, this does not mean that the scattering length *a* is meaningless for fermions: via  $U_c(\vec{r}) = g\delta(\vec{r})$  with  $g = 4\pi\hbar^2/ma_s$  it still enters the Hamiltonian that describes the system.

# 2.3 THE DIPOLAR INTERACTION

Two particles with magnetic dipole moments  $\mu_1, \mu_2$  along the unit vectors  $\vec{e}_1, \vec{e}_2$  and at *Dipole-dipole* relative position  $\vec{r} = \vec{r}_2 - \vec{r}_1$  experience a potential associated to the dipole-dipole interaction (DDI) that can be written as

$$U_{\rm d}(\vec{r}) = \frac{C_{\rm d}}{4\pi} \, \frac{(\vec{e}_1 \cdot \vec{e}_2)r^2 - 3(\vec{e}_1 \cdot \vec{r})(\vec{e}_2 \cdot \vec{r})}{r^5} \qquad \text{with} \qquad C_{\rm d} = \mu_0 \mu_1 \mu_2, \tag{22}$$

where  $\mu_0$  is the vacuum permeability [282].

In experiments with ultracold dipolar atoms, often an external magnetic field is applied. Such a bias field polarises the atoms in the sample ( $\vec{e}_1 = \vec{e}_2$ ) and the dipole potential simplifies to

$$U_{\rm d}(\vec{r}) = \frac{C_{\rm d}}{4\pi} \frac{1 - 3\cos^2\vartheta}{r^3},$$
(23)

Feshbach resonances

Polarised sample

<sup>1</sup> For subtleties about the regularity of such a potential, see Ref. [73] and references therein.

<sup>2</sup> For background on Feshbach resonances see the review of Chin et al. [61], and references therein.



where  $\vartheta$  is the angle between  $\vec{r}$  and the direction of polarisation (cf. Fig. 2).

FIGURE 2: Dipole-dipole interaction between two atoms polarised along  $\vec{B}$ . Depending on the angle  $\vartheta$  (°), the DDI can be either repulsive (red), attractive (blue), or nought (dashed lines). The colour code in (a) is truncated for visual clarity. Figure modified from Ref. [16, § 3.1]

The DDI features some important differences from the contact interaction discussed in Long-range in 3D The previous section. First, due to the  $r^{-3}$ -dependence the DDI is long-range in 3D, but short-range in 2D and 1D. Second, as we see from Eq. 18, the scattering phase shifts for n = 3 scale as  $\delta_{\ell}(k) \propto k$  independent of  $\ell$ . Therefore, all partial waves contribute to the scattering amplitude attributed to the dipolar interaction, independent of temperature. In particular, this has the consequence that identical dipolar fermions can scatter even at  $k \rightarrow 0$ , allowing to cool them via direct evaporation [2]. Third, within  $\vartheta \in [0, \pi/2]$  the factor  $(1 - 3\cos^2 \vartheta)$  varies between -2 and 1, so the interaction potential changes sign. This is intuitively clear, since the dipoles should attract when they are head to tail, and repel when they are side by side. At the magic angle  $\vartheta_{\rm m} = \arccos(3^{-1/2}) \approx 55^{\circ}$ , attraction and repulsion cancel and the DDI vanishes.

Despite the long-range character of the DDI in 3D, we can associate a length scale to it, the so-called dipolar length [163, 254]:

$$a_{\rm d} = \frac{C_{\rm d}m}{12\pi\hbar^2} \tag{24}$$

As we will see, the dipolar length is an important characteristic for deciding whether a sample will behave dominantly dipolar or not.

# 2.4 SCATTERING OF DIPOLAR ATOMS

In general, a dipolar atom creates both, a dipolar as well as a contact potential for another atom. The relative strength between the two is quantified by the ratio

$$\epsilon_{\rm d} = \frac{a_{\rm d}}{a_{\rm s}} = \frac{C_{\rm d}}{3g}.$$
(25)

In principle,  $\epsilon_d > 1$  is required for a sample to behave predominantly dipolar, however, even at  $\epsilon_d \ll 1$  dipolar effects can already become non-negligible [254].

It is not at all obvious and has been intensely debated whether when two particles interact via both, contact and dipolar interaction, the two contributions can be treated separately and the approximation of the contact interaction by a pseudo-potential remains valid. For the most relavant experimental parameter regions, however, away from so-called shape resonances, it is now the accepted opinion that this is the case [163, 300, 301].

At temperatures low enough, in the universal regime [2], the scattering cross section depends not on the particle energy but only on the dipolar length and – for bosons – on the s-wave scattering length. Even though the DDI is anisotropic, we can calculate the mean scattering cross-section of a particle in this universal regime. The 3D average over the full solid angle yields

$$\bar{\sigma} = \begin{cases} \frac{32}{45}\pi a_{\rm d}^2 + 8\pi a_{\rm s}^2 & \text{for bosons,} \\ \frac{32}{15}\pi a_{\rm d}^2 & \text{for fermions,} \end{cases}$$
(26)

wherefrom we see that in case of small s-wave scattering length, fermions scatter about three times more [104].

*Es lohnt sich doch, ein wenig lieb zu sein Und alles auf das Einfachste zu schrauben.* 

- Joachim Ringelnatz [246]

## MEAN-FIELD AND BEYOND

We are now about to merge the insights from the previous chapters and allow particle interactions in our many-body system. If we would naïvely try to build such a description bottom-up, keeping track of the interactions of every particle with everyone else, we would quickly be overwhelmed even at moderate particle numbers – not to mention the tens to hundreds of thousands of atoms in a typical BEC. Instead, such a problem is typically tackled in a mean-field manner.

Generally, a mean-field theory is the approach to study the behaviour of a complex system by considering a much simpler system that approximates the original by averaging over (typically many) degrees of freedom [57]. In our example, we may approximate the system of interacting particles by a system of free particles moving in an effective 'external potential' created by the others. Thus we reduce a computationally hard many-body to a computationally easier, effective one-body problem. However, this intrinsically neglects that each particle locally modifies the potential, i.e. the effect of fluctuations.

As we will see, in the case of a dipolar quantum gas, this deficit can be repaired by simply adding a beyond-mean-field correction term derived using a local density approximation (LDA). Importantly, the result is still a mean-field equation, allowing for reasonably fast computations. Nonetheless, the beyond-mean-field correction is essential for understanding certain intriguing phenomena that can be observed in dipolar quantum gases, such as the formation of quantum droplets or supersolid phases.

## 3.1 THE GPE FRAMEWORK

In mean-field theory, it is possible for ground-state bosons<sup>1</sup> to construct a non-linear Schrödinger equation that contains the interactions at a mean field level. We embark this venture by considering the Heisenberg picture, where state vectors are static and the time evolution of any observable is given by the Heisenberg equation of motion:

$$i\hbar \frac{d}{dt} \hat{O}_{\rm H} = \left[ \hat{O}_{\rm H}, \hat{H}_{\rm H} \right] + \left( i\hbar \frac{\partial}{\partial t} \hat{O} \right)_{\rm H} \tag{27}$$

Here,  $\hat{O}_{\rm H}(t) = \hat{U}^{\dagger}(t)\hat{O}\hat{U}(t)$  and  $\hat{O}$ , respectively, are the Heisenberg- and Schrödingerpicture operators.

We first try to find the right Hamiltonian. For a dilute sample ( $\S$  2.1), we can safely neglect contributions from higher than pairwise interactions, such that

$$\hat{H} = \hat{H}^{(1)} + \hat{H}^{(2)}$$
 with  $\hat{H}^{(i)} = \int \hat{\psi}^{\dagger}(\vec{r}) \hat{h}^{(i)}(\vec{r}) \hat{\psi}(\vec{r}) d^d r.$  (28)

Heisenberg equation of motion

<sup>1</sup> Unfortunately, there is no general analogue of the Gross-Pitaevskii equation for degenerate fermions.

One-body Hamiltonian

Two-body

Hamiltonian

The single-particle Hamilton operator density is simply

$$\hat{h}^{(1)}(\vec{r}) = \underbrace{-\frac{\hbar^2}{2m}\nabla^2}_{\text{kin. energy}} + \underbrace{U^{(1)}(\vec{r})}_{\text{ext. pot.}} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\sum_{i=1}^d \omega_i^2 r_i^2,$$
(29)

with the example of a *d*-dimensional harmonic trap of frequencies  $\omega_i$  given on the righthand side.

The Hamilton operator density  $\hat{h}^{(2)}$  accounts for pairwise interaction. If the interaction potential is short-ranged as in the van der Waals case, it may be replaced by the pseudo-potential (19). Then, away from shape resonances, the pseudo- and long-range potential can be treated separately [163] and we can write:

$$\hat{h}^{(2)}(\vec{r}) = \frac{1}{2} \int \hat{\psi}^{\dagger}(\vec{r}') U^{(2)}(\vec{r} - \vec{r}') \hat{\psi}(\vec{r}') d^d r' \quad \text{with} \quad U^{(2)}(\vec{x}) = U_c(\vec{x}) + U_d(\vec{x}) \quad (30)$$

Note that both the contact ( $U_c$ ) and the dipolar term ( $U_d$ ) can be either positive or negative, and in particular they can be of opposite sign and partially (or completely) cancel each other.

We now apply the Heisenberg equation of motion to the field operator  $\hat{\psi}_{\rm H}(\vec{r})$ . For a Schrödinger-picture field operator  $\hat{\psi}(\vec{r})$  without explicit time dependence, Eq. 27 simplifies to  $i\hbar d_t \hat{\psi}_{\rm H}(\vec{r},t) = [\hat{\psi}_{\rm H}(\vec{r},t), \hat{H}_{\rm H}]$ . Using the commutation relations<sup>2</sup> for the field operators it follows

$$i\hbar \frac{d}{dt}\hat{\psi}_{\rm H}(\vec{r},t) = \left(\hat{h}_{\rm H}^{(1)}(\vec{r},t) + 2\hat{h}_{\rm H}^{(2)}(\vec{r},t)\right)\hat{\psi}_{\rm H}(\vec{r},t).$$
(31)

This expression holds very generally, but since all single-particle states are still coupled through the field operators (Eq. 7), it is quite costly to solve and a greater simplification is desirable for practical use.

*Interacting BECs* Not only the ideal Bose gas, but also the interacting Bose gas undergoes condensation at sufficiently high phase-space density. This can be shown rigorously in 3D [87, 106, 177]; in lower dimensions, at least quasi-condensation occurs with phase coherence over the experimentally relevant length scales (yet not truly long-range) [99, 128, 199]. The fact that the condensate corresponds to the only eigenstate of the single-particle density matrix that has a macroscopic eigenvalue motivates, for a first approximation, to leave all excited states aside.

To do so, we insert the Bogoliubov approximation (11) for all field operators appearing in Eq. 31 (i.e., also inside the Hamiltonians) and neglect the fluctuations  $\hat{\delta}(\vec{r})$ . With this procedure, the operator equation (31) becomes a field equation, which we reflect in the notation by dropping the hats:

$$\hat{\psi}(\vec{r}) \to \Psi(\vec{r}), \quad \hat{h}(\vec{r}) \to h(\vec{r}), \quad \hat{H} \to H.$$
(32)

<sup>2</sup> These lead, in particular, to the factor of two in front of  $\hat{h}^{(2)}(\vec{r})$ , cf. Refs [72, 234, 290].

We further introduce as a short-hand notation the dipolar mean-field potential

$$\Phi(\vec{r}) = \int U_{\rm d}(\vec{r} - \vec{r}') |\Psi(\vec{r}')|^2 {\rm d}^d r'.$$
(33)

The resulting field equation, originally derived in 1961 separately by Gross [125] and Pitaevskii [233] for non-dipolar bosons, then reads

$$i\hbar \frac{d}{dt} \Psi_{\rm H}(\vec{r},t) = \left(h_{\rm H}^{(1)}(\vec{r},t) + 2h_{\rm H}^{(2)}(\vec{r},t)\right) \Psi_{\rm H}(\vec{r},t)$$

$$= \left(\underbrace{-\frac{\hbar^2 \nabla^2}{2m}}_{\rm kinetic} + \underbrace{U^{(1)}(\vec{r})}_{\rm trap} + \underbrace{g|\Psi_{\rm H}(\vec{r},t)|^2}_{\rm contact} + \underbrace{\Phi_{\rm H}(\vec{r},t)}_{\rm dipolar}\right) \Psi_{\rm H}(\vec{r},t)$$
(34)

This equation is called the time-dependent Gross–Pitaevskii equation (GPE). Although it can strictly only be applied in the low-T and weakly interacting limit, it has proven to be a very powerful tool in the description of various types of condensates [72, 149, 172, 223, 234]. Inserting the Heisenberg order parameter (12) into the time-dependent GPE (34), we obtain the time-independent GPE,

$$\mu \Psi(\vec{r}) = \left( -\frac{\hbar^2 \nabla^2}{2m} + U^{(1)}(\vec{r}) + g |\Psi(\vec{r})|^2 + \Phi(\vec{r}) \right) \Psi(\vec{r}).$$
(35)

# Superfluid hydrodynamics

There is a deep connection between the mean-field theory of a Bose–Einstein condensate (BEC) and hydrodynamic behaviour. To see this, we write the time-dependent order parameter in the Madelung form,  $\Psi_{\rm H}(\vec{r},t) = \sqrt{n_0} e^{iS}$ , where density and phase are spaceand time-dependent fields,  $n_0(\vec{r},t)$ ,  $S(\vec{r},t)$ . Inserting this form into the GPE yields the equations of motion for density and phase (Eqs 36 and 37 below, respectively). These two equations are exactly equivalent to the "two" equations given by the GPE (34) for the real and imaginary field,  $\Psi(\vec{r})$  and  $\Psi^*(\vec{r})$  [194, 223].

The equation related to the density reads

$$\frac{\partial n_0}{\partial t} = -\frac{\hbar}{m} \vec{\nabla} (n_0 \vec{\nabla} S) \equiv -\vec{\nabla} (n_0 \vec{v}).$$
(36)

This is a continuity equation that essentially embodies conservation of particle number. Moreover, in the second step, we have introduced the superfluid velocity field which depends on the phase gradient,  $\vec{v} = (\hbar/m)\vec{\nabla}S$ . By definition, this velocity field is irrotational since  $\vec{\nabla} \times (\vec{\nabla}S) \equiv 0$  for all *S*.

Also the equation related to the phase *S* can conveniently be written in terms of the superfluid velocity [194, 234]:

$$m\frac{\partial \vec{v}}{\partial t} = -\vec{\nabla} \left( \frac{1}{2}mv^2 + U^{(1)} + gn_0 + \Phi_d - \frac{\hbar^2}{2m} \frac{(\nabla^2 \sqrt{n_0})}{\sqrt{n_0}} \right)$$
(37)

Stationary GPE

Mean-field GPE

Continuity equation

Superfluid velocity

Euler equation

The last term on the right stems from the zero-point kinetic energy of the particles and has no classical analogue. It is called the quantum pressure term; in the Thomas–Fermi regime, where zero-point motion is small compared to interactions and external potential, this term can be dropped. The equation (37) then takes the same form as the Euler equation for the potential flow of a classical, inviscid fluid.

Note that in the derivation of Eqs ( $_{36-37}$ ) we have made no assumptions except the existence of a classical field  $\Psi(\vec{r})$  that obeys the Heisenberg equation of motion through an equation like the GPE. From this already arises the relation between any quantum condensation phenomenon and superfluidity, regardless of the precise system.

# 3.2 (IN)STABILITIES

From construction we know that the GPE describes a bosonic condensate state. But for which kind of parameters can such a state exist?

To answer this, we start from the full Hamiltonian (28), which in mean-field approximation directly reduces to the Gross–Pitaevskii energy functional,

$$E[\Psi(\vec{r})] = H^{(1)} + H^{(2)}$$

$$= \int \left( \underbrace{-\frac{i\hbar}{2m} |\vec{\nabla}\Psi(\vec{r})|^{2}}_{\mathcal{E}_{K}(\vec{r})} + \underbrace{U^{(1)}(\vec{r}) |\Psi(\vec{r})|^{2}}_{\mathcal{E}_{P}(\vec{r})} + \underbrace{\frac{g}{2} |\Psi(\vec{r})|^{4} + \frac{1}{2} \Phi(\vec{r}) |\Psi(\vec{r})|^{2}}_{\mathcal{E}_{I}(\vec{r})} \right) d^{d}r \quad (38)$$

$$\equiv E_{K} + E_{P} + E_{I}$$

The total mean-field energy for a given field  $\Psi(\vec{r})$  is, accordingly, given by the kinetic energy  $E_{\rm K}$ , trap potential  $E_{\rm P}$ , and interaction energy  $E_{\rm I}$ , which are the integrals over the corresponding energy densities, denoted in cursive font. A scaling analysis in *d* dimensions reveals, for the case of harmonic trapping,

$$E_{\rm K} \propto \frac{N}{L^2}, \qquad E_{\rm P} \propto NL^2, \qquad E_{\rm I} \propto \bar{g} \frac{N^2}{L^d},$$
(39)

where *N* is the condensate atom number, *L* is the mean characteristic size of the cloud and  $\bar{g}$  is the averaged interaction strength (in our case, contact plus dipolar interaction). We find, in the case of three dimensions (*d* = 3):

- For average repulsive interactions ( $\bar{g} > 0$ ), there is always a global minimum in *E* (stable solution) at finite *L*;
- For attractive interactions ( $\bar{g} < 0$ ), the global minimum is at L = 0 since  $E_{I} \rightarrow -\infty$ , leading to a runaway collapse of the condensate if we wait long enough;
- In non-uniform systems ( $E_{\rm K} > 0$ ), even for attractive interactions ( $\bar{g} < 0$ ) there still exists a local minimum (metastable solution) at finite *L* for particle numbers *N* below a critical value  $N_{\rm c}$ .

The energetic instability encountered here affects the whole BEC and is therefore associated with a length scale on the order of L. This corresponds to low-momentum excitations, i. e., as we will see later (§ 3.4), the phonon branch of the excitation spectrum. For this reason, this type of instability is also called a phonon instability.

Energy functional

Phonon instability

In the case of isotropic interactions, such as s-wave scattering,  $E_{\rm K}$  does not depend on the system geometry, allowing to calculate a stability diagram in a straight-forward fashion. For anisotropic interactions, such as the dipolar interaction (§ 2.3), the case is more complex [28, 288, 289]. Let us call the polarisation direction of the dipoles *z*. Without going into details [38, 157, 162, 163, 254], we can then state that in an oblate, 'pancake-shaped' BEC with  $L_z < L_x, L_y$ , the dipoles are on average more side-by-side and the DDI acts dominantly repulsive, stabilising the cloud. In a prolate, 'cigar-shaped' BEC with  $L_z > L_x, L_y$ , in contrast, the dipoles are on average more head-to-tail and the DDI acts dominantly attractive, destabilising the cloud.

In contrast to non-dipolar systems, the aspect ratio  $\kappa_{x,y} = L_{x,y}/L_z$  of a harmonically trapped dipolar BEC is not directly given by the trap aspect ratio  $\lambda_{x,y} = \omega_z/\omega_{x,y} = (\ell_{x,y}/\ell_z)^2$ , where  $\omega_i$  is the trap frequency along direction *i* and  $\ell_i = \sqrt{\hbar/m\omega_i}$  is the associated oscillator length. Following the tendency to minimise the system energy, the dipoles will try to line up along *z*, resulting in  $\kappa_{x,y} < \sqrt{\lambda_{x,y}}$  [83, 209, 210]. This effect is called magnetostriction.

# Stationary solutions

With the GPE we already have the equation that governs the time-evolution of a given field  $\Psi(\vec{r})$ . So far, however, we have no idea what such a state  $\Psi(\vec{r})$  may look like. In particular, we are interested in the shapes of the condensate  $\Psi(\vec{r})$  which for a given trap and scattering length  $a_s$  present stationary solutions to the equations of motion given through the GPE. In general, such (meta-)stable states can correspond to (local) global extrema or saddle points in the associated condensate energy.

For experiments, the states corresponding to energy minima are most relevant. They can be found by minimising the energy functional  $E[\Psi(\vec{r})]$  with respect to  $\Psi(\vec{r})$  at constant condensate particle number  $N_0 = \int |\Psi(\vec{r})|^2 d^d r$ . We can deal with this constraint by interpreting  $\mu$  as a Lagrange multiplier and looking for the  $\Psi(\vec{r})$  that makes the Lagrange function stationary,  $\delta(E - \mu N_0) = 0$ . This procedure is equivalent to minimising  $H^g \Psi(\vec{r})$  at constant  $\mu$ , since in the grand-canonical Hamiltonian,  $H^g = H - \mu N$ , the chemical potential takes care of the particle number [179, 223].

In general (for arbitrary external potentials, etc.), the minimisation has to be performed numerically, starting with a guess for the shape and then performing a gradient descent. In selected cases, e.g. under the Thomas–Fermi approximation where the kinetic energy is neglected, it is possible to obtain an analytical expression for the ground-state  $\Psi(\vec{r})$ .

An alternative way to find a  $\Psi(\vec{r})$  that corresponds to a (local) minimum of  $H^g$  is to evolve a guessed shape with imaginary time (d $t \rightarrow -idt$ ) by integrating Eq. 34 and renormalising after each time step. Since the lowest-energy mode decays the slowest, a good guess will converge towards the ground state.

If the energy functional for a given set of experimental parameters does not possess a nontrivial minimum at finite energy, e. g. when  $\bar{g} < 0$  and  $N_0 > N_c$ , there is no stable BEC. Such a system is (formally) bound to collapse and shrink towards a zero-size and infinite-density object; in an experiment, of course, at some point three-particle scatterings (which we have neglected in our formalism) would become relevant, leading to recombination processes and particle loss.

Magnetostriction

Imaginary-time evolution

Collapse

Non-locality

A final comment on the simulation of dipolar systems. The long-range character of the DDI makes the second term in  $U^{(2)}$  non-local [163, 254]. Therefore, the extended GPE is an integro-differential equation and computationally much more costly to solve than a 'usual' GPE with purely short-ranged interactions.

# 3.3 EXCITATIONS: MANY TIMES BOGOLIUBOV

The GPE describes the dynamics of a BEC. This description is particularly insightful in terms of the eigenmodes of the system. In the laboratory, one often encounters certain types of excitations, such as breathing or sloshing modes. From the fact that a condensate behaves like a fluid ( $\S$  3.1), one might also expect excitations related to sound waves.

The concrete spectrum of excitations can be calculated right away using Bogoliubov theory. For dipolar systems, this was first demonstrated by Santos et al. [255], and in the following we will sketch the general procedure, adapting mainly the formulation of Lima and Pelster [178, 179]. This will provide a framework for the kinds of excitations mentioned above, and reveal that there can be also more exotic kinds, like the roton.

To start, we consider again the field operator in Bogoliubov approximation,  $\hat{\psi}(\vec{r}) = \Psi(\vec{r}) + \hat{\delta}(\vec{r})$ . To relax the constraint of constant particle number right away, we insert it directly into the grand-canonical Hamiltonian,

$$\hat{H}^{g} = \hat{H} - \mu \hat{N} = \int \hat{\psi}^{\dagger}(\vec{r}) \left( h^{(1)}(\vec{r}) + h^{(2)}(\vec{r}) - \mu \right) \hat{\psi}(\vec{r}) d^{d}r$$
(40)

When carrying this out, we separate the contribution of condensate  $\Psi(\vec{r})$  and quantum fluctuations order by order in  $\hat{\delta}(\vec{r})$ . Restricting the expansion to zeroth order one recovers the stationary GPE (35).

We will now go beyond this approximation and retain terms up to second order in fluctuations. We apply the so-called Bogoliubov transformation and expand the fluctuations operator (11) as follows:<sup>3</sup>

$$\hat{\delta}(\vec{r}) = \sum_{i>0} \left( u_i(\vec{r})\hat{b}_i + v_i^*(\vec{r})\hat{b}_i^\dagger \right) \quad \text{with} \quad \int u_i^*(\vec{r})u_j(\vec{r}) - v_i^*(\vec{r})v_j(\vec{r})d^dr = \delta_{ij} \quad (41)$$

Here, the ground state (i = 0) is excluded from the sum and the construction operators  $\hat{b}_i^{\dagger}$ ,  $\hat{b}_i$ , which have no interpretation at this stage, obey the bosonic commutation relations. The resulting Hamiltonian is diagonal if the following pair of coupled Bogoliubov–de Gennes (BDG) equations is satisfied (cf. Refs [32, 107, 250]):

BDG equations

$$\mathcal{L}\begin{pmatrix}u_i\\v_i\end{pmatrix} = \varepsilon_i \begin{pmatrix}u_i\\v_i\end{pmatrix},\tag{42}$$

where

$$\mathcal{L} = \begin{pmatrix} (h^{(1)} + 2h^{(2)} - \mu) + X & -X \\ X & -(h^{(1)} + 2h^{(2)} - \mu) - X \end{pmatrix},$$
(43)

3 By writing Eq. 41, the sign convention of Pitaevskii & Stringari [234] is chosen.

Bogoliubov transformation

Grand-canonical Hamiltonian  $\varepsilon_i$  is the excitation energy of mode *i*, and for compactness the dependence on  $\vec{r}$  is not explicitly noted. The plain  $h^{(2)}$ -terms in Eq. 43 account for the usual direct interaction between two condensate particles. The exchange operator X is defined by

$$Xw_i(\vec{r}) = \Psi(\vec{r}) \int U^{(2)}(\vec{r} - \vec{r}') w_i(\vec{r}') \Psi^*(\vec{r}') d^d r'$$
(44)

with  $w_i \in \{u_i, v_i\}$  and accounts for the exchange interaction between one particle from the condensate and one from an excited state.

The general recipe (42) allows to compute the Bogoliubov amplitudes  $u_i(\vec{r})v_i(\vec{r})$  and the spectrum  $\varepsilon_i$ . In the general case, this has to be carried out numerically; in some special cases, analytical solutions are known. An instructive example is the homogeneous gas outlined in the next section.

# 3.4 SPECTRUM OF A 3D HOMOGENEOUS BEC

The basis states of the homogeneous system are plane waves. The associated plane-wave momenta  $\vec{k}$  are conserved and good quantum numbers [277, 290]. It is therefore convenient to work in momentum space, where in three dimensions, the Fourier transform of the interaction potential  $U^{(2)}(\vec{r})$  (cf. Eq. 30) takes the form

$$\tilde{U}^{(2)}(\vec{k}) = \tilde{U}_{c}(\vec{k}) + \tilde{U}_{d}(\vec{k}) = g(1 + \epsilon_{d}(3\cos^{2}\vartheta - 1)).$$
(45)

In a homogeneous system, the condensate field  $\Psi$  must be independent of position, so the mean-field chemical potential has to take the form  $\mu = n_0 \lim_{k\to 0} \tilde{U}^{(2)}(\vec{k})$ , where  $n_0 = N_0/V$  is the number density in a volume *V*. This expression can be inserted into the BDG equations (42), finally allowing to solve (i) for the Bogoliubov amplitudes [179], labelled by (discrete) momenta  $\vec{k}$ ,

$$v_{\vec{k}}^2 = \frac{1}{2\varepsilon_{\vec{k}}} \left( \frac{\hbar^2 k^2}{2m} + n_0 \tilde{U}^{(2)}(\vec{k}) \right) - \frac{1}{2},\tag{46}$$

through which by the Kronecker condition (41) also the  $u_{\vec{k}}$  are fixed, as well as (ii) for the Bogoliubov spectrum [179, 256]:

Bogoliubov spectrum

$$\varepsilon_{\vec{k}} = (\pm) \sqrt{\frac{\hbar^2 k^2}{2m} \left(\frac{\hbar^2 k^2}{2m} + 2gn_0 \left(1 + \epsilon_d (3\cos^2\vartheta - 1)\right)\right)}$$
(47)

where  $\vartheta$  is the angle between  $\vec{k}$  and the dipole orientation. Since  $\varepsilon(k)$  is either real or purely imaginary, we can restrict ourselves to positive solutions. Negative mode energies would simply swap the roles of the ladder operators  $\hat{b}_{\vec{k}}^{\dagger}, \hat{b}_{\vec{k}}$  and are hence not required for the completeness of the Bogoliubov eigenbasis [106, 223].

Let us inspect the low- and high-momentum limits of the spectrum (47):

$$\varepsilon_{\vec{k}} \to \begin{cases} c_1(\vartheta) \, k & \text{for } k \to 0 \quad \text{(phonon-like),} \\ \frac{\hbar^2}{2m} \, k^2 & \text{for } k \to \infty \quad \text{(free-particle-like),} \end{cases}$$
(48)

where

$$c_1(\vartheta) = c_1^0 \sqrt{1 + \epsilon_d(\cos^2 \vartheta - 1)} \quad \text{and} \quad c_1^0 = \sqrt{gn_0/m}.$$
(49)

Here,  $c_1^0$  is the velocity of first sound in the non-dipolar limit ( $\epsilon_d \ll 1$ ).

At low momenta (or long wavelengths), the excitations are collective and propagate like sound waves. For dipolar gases, the speed of sound  $c_1(\vartheta)$  depends on direction due to the anisotropy of the dipolar interaction, as has been verified in experiments in Paris [27] and Stuttgart [295]. Moreover, for  $\epsilon_d > 1$  there are angles  $\vartheta$  where  $c_1(\vartheta)$  – and hence the mode energy  $\varepsilon_{\vec{k}}$  – can become imaginary. This dynamic instability on the phonon branch connects directly to the energetic instability discussed in § 3.2. Instabilities will be revisited in greater detail in § 3.6, where we will take a look at (i) finite systems, more realistic from an experimental perspective, and will (ii) put a special focus on the regime between the low- and the high-*k* branch. Here, for dipolar systems, a rotonic dispersion can be observed.

At high momenta (i. e., short wavelengths), the excitations of a  $_{3}D$  homogenous system are quadratic in k and again resemble the excitations of free particles.

#### Bogoliubov excitations as Nambu–Goldstone bosons

In a homogeneous system, the Hamiltonian diagonalised through the BDG approach takes a particularly simple form:

$$\hat{H} = E_0 + \sum_{k>0} \varepsilon_{\vec{k}} \hat{b}^{\dagger}_{\vec{k}} \hat{b}_{\vec{k}} \tag{50}$$

Here,  $E_0 = \hat{H} |G_N\rangle$  is the *N*-body ground-state energy<sup>4</sup> of the condensate and the ladder operators  $\hat{b}_{\vec{k}}^{\dagger}, \hat{b}_{\vec{k}}$  add or remove Bogoliubov excitations of energy  $\varepsilon_{\vec{k}}$ .

As discussed in § 1.4, the classical field  $\Psi(\vec{r})$  acts as an order parameter for Bose–Einstein condensation. The thermal phase has a U(1) symmetry, which is broken when crossing the phase transition to the condensate and  $\Psi(\vec{r})$  spontaneously takes a particular phase value. By the Goldstone theorem [111], every symmetry breaking implies the existence of excitations related to the broken symmetry, the so-called Nambu–Goldstone bosons. The Bogoliubov excitations are the Nambu–Goldstone bosons of Bose–Einstein condensation [106, 110, 174, 204]. The operators  $\hat{b}^{\dagger}_{\vec{k}}, \hat{b}_{\vec{k}}$  are the corresponding quasiparticle creators and annihilators, for whom the *N*-body ground state acts as the vacuum,  $\hat{b}_k |G_N\rangle = 0$ .

Anisotropic speed of sound

Phonon instability

<sup>4</sup> Including a beyond-mean-field correction, cf. § 3.5.2 and Ref. [234].

#### 3.5 QUANTUM FLUCTUATIONS IN LOCAL-DENSITY APPROXIMATION

Non-uniform systems, as in typical experiments, are harder to treat than the homogeneous case. A popular approach is to make a local density approximation (LDA), where the number density  $n(\vec{r})$  is allowed to vary in space, but in each point, the calculation is performed for a homogeneous system of volume *V* and density  $N/V = \text{const.} = n(\vec{r})$  [28, 178, 179, 289]. Such a procedure leads to important insights, as will be presented in the following.

## 3.5.1 Quantum depletion

The number of particles in the many-body ground state  $|G_N\rangle$  is given by the expectation value  $\langle G_N | \hat{N} | G_N \rangle$ . For a diagonal Hamiltonian [178, 179], the particle density and total particle number separate into

$$n(\vec{r}) = |\Psi(\vec{r})|^2 + \langle \hat{\delta}^{\dagger}(\vec{r})\hat{\delta}(\vec{r}) \rangle = n_0(\vec{r}) + \Delta n(\vec{r}) \quad \text{and}$$
(51)

$$N = \int n(\vec{r}) \mathrm{d}^d r = N_0 + \Delta N, \tag{52}$$

respectively.  $N_0$  is the number of particles in the single-particle ground state (k = 0). We see that when interactions are present and  $\langle \hat{\delta}^{\dagger} \hat{\delta} \rangle \neq 0$ , even in a system in the *N*body ground state  $|G_N\rangle$  at T = 0, a finite number  $\Delta N$  of particles is depleted from the single-particle ground state (described by  $\Psi$ ) and occupies excited single-particle states. The depleted particles are described by the fluctuations operator  $\hat{\delta}$ , such that we obtain in local-density approximation<sup>5</sup> [32, 179]:

$$\Delta n(\vec{r}) = \langle \hat{\delta}^{\dagger}(\vec{r}) \hat{\delta}(\vec{r}) \rangle = \left\langle \sum_{k,k'>0} \left( u_{\vec{k}}^{*}(\vec{r}) \hat{b}_{\vec{k}}^{\dagger} + v_{\vec{k}}(\vec{r}) \hat{b}_{\vec{k}} \right) \left( u_{\vec{k}'}(\vec{r}) \hat{b}_{\vec{k}'} + v_{\vec{k}'}^{*}(\vec{r}) \hat{b}_{\vec{k}'}^{\dagger} \right) \right\rangle$$
  
$$= \sum_{k>0} \int v_{\vec{k}}^{*}(\vec{r}) v_{\vec{k}}(\vec{r}) d^{d}r$$
(53)

Here, cross-terms vanish due to the commutation relations of  $\hat{b}^{\dagger}_{\vec{k}}$ ,  $\hat{b}^{\dagger}_{\vec{k}}$  (cf. Eqs 111–112) and the Kronecker condition (41).

In the thermodynamic limit, the spectrum discrete in  $\vec{k}$  becomes continuous such that we can replace the sum by an integral,  $\sum_{k>0} \rightarrow V(2\pi)^{-3} \int d\vec{k}$ , and insert the expression for the Bogoliubov modes obtained earlier for the homogeneous system (Eq. 46). This yields

$$\Delta n(\vec{r}) = \frac{8}{3\sqrt{\pi}} \mathcal{Q}_3(\epsilon_{\rm d}) n^{1/2}(\vec{r}) a_{\rm s}^{3/2}$$
(54)

[178, 179], where the auxiliary function is defined through

$$Q_{\ell}(x) = \int_0^1 (1 - x + 3xy^2)^{\ell/2} dy.$$
(55)

<sup>5</sup> NB that without LDA, momentum is not a good quantum number for an inhomogeneous system and we would not be able to label the Bogoliubov modes by  $\vec{k}$ .

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This function increases monotonically with dipolarity  $\epsilon_d$  from  $Q_3(0) = 1$  for a purely contact-interacting system to  $Q_3(1) \approx 1.30$  for a fully dipolar system.<sup>6</sup> Compared to nondipolar systems, quantum depletion is therefore increased in dipolar systems; however, for typical experimental parameters the depleted fraction is still on the low percent level, justifying the theoretical treatment of the depleted fraction as a perturbation to the condensate. This is in stark contrast to *strongly interacting* systems such as liquid helium, where the range of interparticle interactions is on the order of the interparticle separation, leading to a depleted fraction typically higher than 90 % [270, 271, 276]. Consequently, perturbation theory fails for superfluid helium and more advanced approaches like Monte Carlo or variational methods are necessary.

For arguments x > 1, the function  $Q_{\ell}(x)$  takes imaginary values. This marks another type of instability, as will be revisited a bit later (§ 3.6.2).

## 3.5.2 The LHY energy correction

Also the energy of the condensate is affected by the presence of quantum fluctuations. In LDA for harmonic trapping, Eq. 38 takes the form  $E = E_{\rm K} + E_{\rm P} + E_{\rm I} + \Delta E$ , where the energy correction  $\Delta E = \int \Delta \mathcal{E}(\vec{r}) d^d r$  is within Bogoliubov theory [28, 179] given by the energy density

$$\Delta \mathcal{E}(\vec{r}) = \frac{2}{5} \gamma \, n^{5/2}(\vec{r}) \tag{56}$$

with the quantum fluctuation parameter

$$\gamma = \frac{32}{3\sqrt{\pi}} \mathcal{Q}_5(\epsilon_{\rm d}) g \, a_{\rm s}^{3/2} = \frac{32}{3\sqrt{\pi}} g \, a_{\rm s}^{3/2} \left( 1 + \frac{3}{2} \epsilon_{\rm d}^2 + \mathcal{O}(\epsilon_{\rm d}^4) \right). \tag{57}$$

In this case, the auxiliary function varies from  $Q_5(0) = 1$  (purely contact-interacting) to  $Q_5(1) \approx 2.60$  (dipolar system).

For homogeneous density and a purely contact-interacting gas ( $\epsilon_d = 0$ ), one recovers the 1957 seminal result of Lee, Huang & Yang [170], wherefore the energy correction (56) is referred to as the LHY correction.

The LDA energy correction (56) further results in a *local* correction to the chemical potential [28, 178, 179, 289]:

$$\Delta\mu(\vec{r}) = \frac{\partial(\Delta\mathcal{E})}{\partial n}(\vec{r}) = \gamma \, n^{3/2}(\vec{r}) \tag{58}$$

# 3.5.3 The extended Gross-Pitaevskii equation

As we have seen, even though the quantum fluctuations  $\hat{\delta}(\vec{r})$  are a genuine *beyond-mean-field* effect, under the local-density approximation their effect can to a certain extent be harnessed in form of *mean-field* correction terms  $\Delta \mathcal{E}(\vec{r}), \Delta n(\vec{r}), \Delta \mu(\vec{r})$ . Continuing in this

<sup>6</sup> For a plot of  $Q_3(x)$  and  $Q_5(x)$ , see, e.g., Refs [178, 226].

spirit, the effect of quantum fluctuations can be woven into the Gross–Pitaevskii equation (34) that describes the system, in time-dependent as well as in stationary form:

$$i\hbar \frac{d}{dt} \Psi_{\rm H}(\vec{r},t) = \left( -\frac{\hbar^2 \nabla^2}{2m} + U^{(1)}(\vec{r}) + g |\Psi_{\rm H}(\vec{r},t)|^2 + \Phi_{\rm H}(\vec{r},t) + \gamma |\Psi_{\rm H}(\vec{r},t)|^3 \right) \Psi_{\rm H}(\vec{r},t)$$
(59)

$$\mu \Psi(\vec{r}) = \left( -\frac{\hbar^2 \nabla^2}{2m} + U^{(1)}(\vec{r}) + g |\Psi(\vec{r})|^2 + \Phi(\vec{r}) + \Delta \mu(\vec{r}) \right) \Psi(\vec{r})$$
(60)

These equations were originally proposed in Refs [28, 179, 289] and are called the extended Gross–Pitaevskii equations.

The corrected energy functional associated with the extended GPE reads, accordingly,

$$E[\Psi(\vec{r})] = \int \left( \mathcal{E}_{\mathrm{K}}(\vec{r}) + \mathcal{E}_{\mathrm{P}}(\vec{r}) + \mathcal{E}_{\mathrm{I}}(\vec{r}) + \Delta \mathcal{E}(\vec{r}) \right) \mathrm{d}^{d}r$$

$$= \int \left( -\frac{\mathrm{i}\hbar}{2m} |\vec{\nabla}\Psi(\vec{r})|^{2} + U^{(1)}(\vec{r})|\Psi(\vec{r})|^{2} + \frac{1}{2m} - \frac{1}{2m}$$

$$+\frac{1}{2}g|\Psi(\vec{r})|^{4}+\frac{1}{2}\Phi(\vec{r})|\Psi(\vec{r})|^{2}+\frac{2}{5}\gamma|\Psi(\vec{r})|^{5}\bigg)d^{d}r.$$
(62)

## 3.6 (IN)STABILITIES REVISITED

Let us now investigate the effects which quantum fluctuations can have on the stability of a dipolar quantum gas. We will see that the dipolar interaction quantum can trigger a roton instability, which in turn can be stabilised through a mechanism stemming from the quantum fluctuations we have encountered in the previous section. In combination, dipolar interactions and quantum fluctuations present the two essential ingredients for the formation of arrays of dipolar quantum droplets and dipolar supersolid phases of matter.

## 3.6.1 A quantum-fluctuation-driven stabilisation mechanism

In 2015 it was realised in a seminal paper by Petrov [225] that a mean-field unstable quantum system could be stabilised by the Lee–Huang–Yang (LHY) term. In his study, a Bose–Bose mixture with a repulsive intraspecies and an attractive interspecies interaction was considered in three dimensions. If the attractive interactions are dominant, such a mixture is expected to collapse according to the energy functional (38) in the zero-order mean-field picture. If, however, the repulsive and attractive interactions can be controlled independently, one can create a situation where they almost cancel and a gentle collapse can be balanced by the repulsive LHY term in the corrected energy functional (cf. Eq. 61) due to its higher scaling in density (cf. Eq. 39):

$$E_{\rm I} \propto \bar{g} \frac{N^2}{L^3}$$
 vs  $\Delta E \propto n^{5/2} L^3 = \frac{N^{5/2}}{L^{9/2}}$  (63)

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Such a system stabilised by quantum fluctuations (QF) shows peculiar properties, unknown at that point from any other ultracold-atom system [225]. For example, due to the steeper scaling in density, it was predicted that a QF-stabilised system could become self-bound such that it can stably exist in vacuum without any trapping. The particle densities in such a QF-stabilised system are one to two orders of magnitude larger than for conventional quantum gases. Even more uniquely, the peak density is expected to saturate, resulting in a flat-top density profile and justifying the name *quantum droplet*, in reminiscence of classical fluid droplets. Also the excitation spectrum of a quantum droplet is peculiar, containing only few or no discrete modes. For untrapped droplets, there are regimes where no collective modes are below the chemical potential, meaning that all excitations must result in a spilling of particles, leading to a self-evaporation of the droplet to zero temperature [44, 225].

It was soon realised after the publication of Petrov, that, similar to independently tunable intra- and interspecies interactions in a mixture, in dipolar Bose gases a balancing of the attractive dipole-dipole interaction and a repulsive contact interaction could allow to access a regime that would favour the formation of self-bound dipolar droplets [17].

These early works have sparked a wealth of following theoretical studies,<sup>7</sup> and have motivated several experimental groups, including ours, to set out and test their enthralling predictions.

## 3.6.2 The dipolar roton

For finite dipolar systems, like an experimental BEC, there is an important difference compared to the excitation spectrum (47) of the infinite 3D homogeneous gas. In an infinite 3D homogeneous system, the Fourier transform of the dipole potential (cf. Eq. 45) depends on the direction ( $\vartheta$ ), but not the modulus of momentum  $\vec{k}$ .

As was realised in the early 2000s [208, 255], if a dipolar system is confined in space, this is not anymore the case. Suppose the system is confined harmonically in the direction z of the dipoles with frequency  $\omega_z$  and oscillator length  $\ell_z$ . Two theoretical models are worthwhile to study in this context: the infinite pancake in the *xy*-plane (Fig. 3d), and the infinite cigar along y (Fig. 3a). From an initial focus on the pancake geometry in early theoretical studies [208, 255] and experiments [145], the focus has later shifted more towards the cigar geometry, which has proved more favourable for implementing experiments directly probing the excitation spectrum through a geometric focussing effect (higher peak value in momentum distribution, cf. Fig. 3c, d).<sup>8</sup>

In either case, momentum along the infinite direction(s), transverse to the confinement, is a good quantum number, and hence plane waves in density form a convenient choice for the corresponding excitations.

For transverse wavelengths  $\lambda_{\perp} = 2\pi/k_{\perp}$  much greater than  $\ell_z$ , the wave propagation is essentially mediated exclusively via side-by-side repulsion between dipoles. These excitations are phonons in an effective 2D geometry (bottom left cartoon in Fig. 3b).

For in-plane wavelenghts  $\lambda_{\perp} \ll \ell_z$ , in contrast, the excitations are of 3D character and include attractive head-to-tail attractions, which lower the excitation energy (bottom right

<sup>7</sup> See, e.g., the review of Böttcher et al. [44], and references therein.

<sup>8</sup> Most recently, with a better understanding of the physics at play and a refined control of experimenal parameters, interest is shifting back towards the pancake geometry [136, 207, 258].



FIGURE 3: Dipolar particles in a confined geometry. (a) An infinite cigar trap with transverse dipole alignment. (b) The corresponding excitation spectrum. The roton mode softens for decreasing as and becomes imaginary (dotted line depicts norm). Cartoons on bottom illustrate dipole configurations for low-(left) and high- $k_y$  (right) momentum. (c, d) Momentum distribution for equal roton mode populations in a cigar (a, c) and pancake (d) geometry, in identical colour scale. Figure modified from Ref. [66].

cartoon in Fig. 3b). As a result, for increasing momentum k, the effective dipolar interaction potential  $\tilde{U}_{d}^{\text{eff}}(k_{\perp})$  decreases and turns negative for  $k_{\perp} > 1/\ell_z$ . This has an important effect on the excitation spectrum, sketched in Fig. 3b.

Let us divide the discussion of this dispersion curve in three distinct regions:

- At low momenta,  $k_{\perp} \rightarrow 0$ , the DDI is dominantly repulsive and leads to a linear phonon dispersion.
- At high momenta,  $k_{\perp} \rightarrow \infty$ , the excitation spectrum is still quadratic and free-particle-like.
- At intermediate momenta, around k<sub>⊥</sub> ~ 1/ℓ<sub>z</sub>, the attractive DDI lowers the excitation energy, i. e. the mode becomes easier to excite and appears 'softer'.

If strong enough, the mode softening around  $k_{\perp} \sim 1/\ell_z$  can lead to the development of a characteristic maximum-minimum sequence, where the maximum and minimum are referred to as maxon and roton<sup>9</sup>, respectively. These names were originally coined in the context of superfluid helium, which exhibits a qualitatively similar dispersion [94, 164, 212, 260, 298]. Around the roton minimum, the dispersion curve can be approximated quadratically. For the infinite cigar, this can be written as [66]

$$\varepsilon(k_y)\Big|_{k_y \approx k_R} \approx \Delta_R^2 + \frac{2\hbar^2 k_R^2}{m} \,\frac{\hbar^2 (k_y - k_R)^2}{2m} \tag{64}$$

<sup>9</sup> This denomination, despite sounding suggestive, does not have an intuitive interpretation connected to a rotation.



FIGURE 4: *The dispersion of a dipolar quantum gas of* <sup>166</sup>*Er atoms.* From left to right, the sample becomes increasingly dipole-dominated, with experimentally determined scattering lengths  $a_s \in \{80.0, 60.5, 55.3, 52.5\}a_0$ , respectively. Plot markers are experimental data from Bragg spectroscopy, the colour code reflects the normalised dynamic structure factor  $\tilde{S}_0(k, \omega)$  predicted by zero-temperature theory. Figure modified from Ref. [227].

with the roton momentum  $k_{\rm R} = \sqrt{2m}(E'^2 - \Delta_{\rm R}^2)^{1/4}$ , the roton gap  $\Delta_{\rm R} = \sqrt{E'^2 - E''^2}$  and energies  $E'^2 = 2gn_0\epsilon_{\rm d}(\hbar^2/2m)(X^{-2} + Z^{-2})$  and  $E'' = (2/3)gn_0(\epsilon_{\rm d} - 1)$ . Here,  $n_0$  is the density in the centre and X, Z are the Thomas–Fermi radii in the *x*- and *z*-direction.

In an experiment with ultracold dipolar atoms, both the roton momentum and the roton gap can be varied. This is achieved, on the one hand, by control of the confinement  $\ell_z$  which determines the Thomas–Fermi radius *Z*, and, on the other hand, via the dipolar parameter  $\varepsilon_d$  which depends inversely on the scattering length  $a_s$ . The scattering length  $a_s$  can be tuned at a magnetic Feshbach resonance. This has been measured directly in our group by Petter et al. [227] for erbium, using Bragg spectroscopy (see Fig. 4), and indirectly by the Pfau group in Stuttgart [134] for dysprosium. In their approach, the Stuttgart group relied on the statistical tools of density correlation and principal-component analysis, as demonstrated by our group [205] and initially pioneered in non-dipolar systems [80, 264].

Dynamic instabilities The possibility to tune the dispersion relation brings about great experimental opportunities. Importantly, for suitable experimental parameters, it is possible to reach the regime  $E'^2 \ge E'^2$ , so the roton gap can vanish and even become imaginary. To see what this entails, we write the fluctuation operator, which embodies all excitations of the system, in Heisenberg form. Recalling that after the Bogoliubov expansion (41) the Hamiltonian is of diagonal shape, application of the time-evolution operator  $\hat{U}(t) = \exp(i\hat{H}t/\hbar)$  simply yields

$$\hat{\delta}_{\mathrm{H}}(\vec{r},t) = \hat{U}^{\dagger}(t)\hat{\delta}(\vec{r})\hat{U}(t) = \sum_{k} \left( u_{k}(\vec{r})\hat{b}_{k}\mathrm{e}^{-\mathrm{i}\varepsilon(k)t/\hbar} + v_{k}^{*}(\vec{r})\hat{b}_{k}^{\dagger}\mathrm{e}^{\mathrm{i}\varepsilon(k)t/\hbar} \right).$$
(65)

For real mode energies  $\varepsilon(k)$ , this presents an oscillation with angular frequency  $\omega_k = \varepsilon(k)/\hbar$ , hence a *stable* mode. However, as we have seen, in the extreme case of strongly dominating dipolar attraction the mode energy can become imaginary. The corresponding exponents in Eq. 65 then turn real and we are faced with an *unstable* mode whose occupation grows exponentially in time.

If this happens at low momentum  $k \rightarrow 0$ , in the phonon branch, the associated length scale is as large as the system, and the instability is of global character; this is the condensate collapse described in § 3.2.





(a) The dispersion relation from Rosensweig's 1985 monograph [251]. The curves give the dispersion for growing influence of the magnetic interaction (along arrow *M*).

(b) A photograph of a classical ferrofluid beyond the Rosensweig instability. Figure modified from Ref. [156].

FIGURE 5: Dispersion and instability of a classical ferrofluid.

If, in contrast, it happens at intermediate momentum, as in the roton case (Eq. 64), the associated length scale is appreciably smaller than the system itself and the instability has a local character. We can try to get an intuition what such a local instability can look like by considering an analogous system from classical physics.

A roton is not a phenomenon restricted to the quantum world. Also classical fluids can show a drastic modification of the dispersion relation if they feature, for example, a sufficient magnetic interaction. This can be demonstrated with so-called ferrofluids, which consist of ferromagnetic nanoparticles suspended in a liquid solution. For ferrofluids, an entirely classical analysis [251] predicts dispersion relations qualitatively similar to the dipolar Bose gas, with a maxon and a roton between the linear (phonon) and the quadratic (free-particle) branch. As illustrated in Fig. 5a, the dispersion of the ferrofluid depends on the sample magnetisation, which can be manipulated through an external field. From complete absence without magnetisation ( $M_0 = 0$ ), the roton mode increasingly softens for growing magnetisation (following the arrow M).

At a critical magnetisation, the roton touches zero (black bullet). Something dramatic happens to the ferrofluid at this "exchange of instabilities" [251]: Beyond this critical point, the ferrofluid can no longer maintain a flat and smooth surface, but spontaneously develops sharp spikes arranged in a regular pattern, as shown in Fig. 5b. Physically, it is the point where the surface tension, trying to minimise the surface area, is overcome by the tendency of the nanoparticles to arrange head to tail instead of side by side. We see from this classical example that the softening of a roton mode can lead to a breaking of the translational symmetry of a system.

A similar instability can occur in dipolar quantum gases, leading to new phases of matter: quantum droplets, dipolar supersolids, and possibly more, still awaiting their discovery, such as dipolar superglasses [136]. Peculiarities of these exotic states will be discussed in the context of the publications presented in §§ 6 sqq. Rotons in classical ferrofluids

*I believe that truth has only one face: that of a violent contradiction.* 

- Georges Bataille [79]

# CAN A SUPERFLUID BE SOLID?

The following chapter is intended to introduce the historic idea of a 'superfluid solid', to provide formal definitions of quantities important in this context, and to review experiments aimed at the detection of supersolids in systems other than dipolar quantum gases.

# 4.1 A SHORT HISTORY OF SUPERPROPERTIES

The advent of superproperties is inseparably tied to the Dutch refrigeration pioneer Kammerlingh Onnes, who, in the early 1900s, established a cryogenic laboratory at Leiden with the aim of testing thermodynamic predictions at low temperature. Already at that time it was known that helium has the lowest boiling point of all materials. As we know now, the reason why helium stays liquid at ambient pressure even for  $T \rightarrow 0$  is the combination of a weak<sup>1</sup> inter-atomic attraction and a large zero-point motion<sup>2</sup> [86]. Unaware of these peculiarities, in 1908 Kammerlingh Onnes succeeded in liquefaction of the more abundant isotope <sup>4</sup>He, mainly driven by the goal to use it as a refrigerant. This enabled the discovery of the abrupt dissappearance of electrical resistance in solid mercury below 4.2 K on 8 April 1911. The immediate excitement about this doubtlessly groundbreaking discovery made him and his coworkers attach only little significance to the fact that, later the same day, when the temperature dropped below 2.17 K, the violent boiling of the helium suddenly stopped and gave way to strong evaporation from a perfectly still surface, signalling a leap in thermal conductivity [77].<sup>3</sup> So on this momentous day, two quantum phase transitions, superconductivity and superfluidity, had been observed for the first time, but the significance of the latter was only realised much later.

A third phase transition was discovered in Leiden in 1926, when Keesom studied liquid helium under externally applied pressure [148]. Differently from room pressure, if the density of helium is increased, the atoms can come close enough for the van-der-Waals force to take over such that solid bonds form and the helium freezes.

In 1938, finally, Kapitsa and, with an independent experiment, Allen and Misener took attempts to study liquid helium below the  $\lambda$ -point more closely. Both of their experiments yielded results consistent with a viscosity of zero [4, 147], and Kapitsa called this "special

<sup>&</sup>lt;sup>1</sup> Caveat: As the lightest noble gas, helium atoms have a particularly small diameter, wherefore orbitals are not easily deformed and induced dipole-dipole attractions are *too weak* for solidification, in contrast to the *metastable* BECs of metallic vapours, for which the true ground state is a solid. However, liquid helium is typically around 8 orders of magnitude more dense than the BECs of metallic vapours, and the mean spacing of helium atoms is not large compared to the interaction range. Therefore, from a quantum-gas perspective it must be regarded as a *strongly-interacting* system.

<sup>2</sup> The zero-point energy scales inversely with the particle mass and is hence more relevant for light elements.

<sup>3</sup> The extraordinary heat conductance stems from the fact that in <sup>4</sup>He below the  $\lambda$ -transition ("helium-II") heat is not transported as usual via diffusion, but via entropy waves, the so-called second sound.

#### 32 CAN A SUPERFLUID BE SOLID?

state" a superfluid, in analogy to the electric superconductors.<sup>4</sup> It took not long until Fritz London drew an analogy between the scattering of fermions leading to electrical resistance and the scattering of bosons leading to fluid viscosity [182]. Intuitively, he conjectured that a process similar to the Bose–Einstein condensation of an ideal gas could prevent particle scattering in helium-II and explain the dissipationless flow [181]. In 1941, finally, Landau suggested a convincing model to explain the absence of viscosity [86, 164, 234]. His important insight was that for the creation of an excitation of wave vector  $\vec{k}$  and energy  $\varepsilon(k)$  to be created from the motion of the fluid, conservation of energy and momentum require

$$\varepsilon(k) + \hbar \vec{k} \cdot \vec{v} \le 0, \tag{66}$$

which can only be fulfilled if the second term is negative and the velocity v is larger than a certain minimum value [234]. The minimum velocity over all wave vectors k for which an excitation can be created in the system is the critical velocity,

$$v_{\rm c} = \min_{k} \frac{\varepsilon(k)}{\hbar k}.\tag{67}$$

Through this relation, the critical velocity  $v_c$  depends on the excitation spectrum  $\varepsilon(k)$  of the system.

In an ideal gas, particles follow the free-particle dispersion,  $\varepsilon(k) = \hbar^2 k^2 / 2m$ , yielding  $v_c = 0$ , allowing momentum transfer at infinitely small velocities and giving rise to dissipation and viscosity. If, in contrast, the dispersion relation is phonon-like,  $\varepsilon(k) = c_1^0 \hbar k$ , where  $c_1^0$  is the (first) sound velocity, then  $\varepsilon(k) / \hbar k = \text{const.} > 0$  at low momentum and the critical velocity  $v_c$  stays finite. As we have seen in Eq. 48, the excitations of an interacting Bose gas at small k are collective, giving rise to a phonon branch, whereas single-particle excitations occur only at high k. This is the reason why interacting BECs are superfluid, whereas the BEC of an ideal gas is not.

Historically, the theory of Bose–Einstein condensation in interacting systems was only rigorously worked out in 1956 by Penrose and Onsager [220], finally setting the speculations of London on firm grounds. This provided a framework which stimulated much following work on superfluidity. A particular question was what happens to the superfluidity of helium at the phase transition from liquid to solid, and whether it could be, in some way maintained [7, 59, 123, 124]. This led Legget to ask "can a solid be 'superfluid'?" in a famous article in 1970, which coined the counter-intuitive notion of *supersolidity* [171].

Motivated by the prospect of discovering an exotic phase of matter, generations of experimentalists have quested after the supersolid. However, as will be explained in §4.3, their search has long been in vain and only very recently started to bear fruit.

In particular, from solid helium, which has long been the prime candidate for a supersolid, priorities have now shifted to the widely tunable ultracold atom systems such as BECs. By this, the question of Legget may rather be rephrased in reverse, asking:

Can a superfluid be 'solid'?

<sup>4</sup> Nowadays, this intuition is rather turned around and superconductors are essentially considered charged superfluids [41].

#### 4.2 FORMAL DEFINITION OF SUPERSOLIDITY

Imagining a macroscopic substance that is solid and at the same time fluid can seem quite puzzling to one's intuition. Before talking about supersolids it is therefore important to precisely define what is meant. In the following, we will recapitulate some key notions about supersolidity. For greater detail, the reader is referred to the comprehensive colloquium by Boninsegni and Prokof'ev [41].

#### Order in a solid

Taking a microscopic view, the time-averaged local number density  $n(\vec{r})$  of a crystalline solid is periodic in space,

$$n(\vec{r}) = n(\vec{r} + \vec{R}) \quad \text{with} \quad \vec{R} = \sum_{i=1}^{d} m_i \vec{a}_i, \quad m_i \in \mathbb{Z},$$
(68)

where the  $\vec{a}_i$  denote the primitive lattice vectors of a *d*-dimensional Bravais lattice. This socalled density long-range order (LRO) effectively constitutes a breaking of the translational symmetry. Of course, in reality, solids are necessarily of finite extension, but it turns out that many of the characteristic macroscopic features of solids are already present for small crystals with  $m_i$  on the order of a few.

It is fundamental for a substance to be called a solid that the density modulation develops spontaneously, as a consequence solely of the interactions between its constituents [41]. If the modulation is externally imposed, as for example for atoms trapped in an optical lattice, it is therefore incorrect to speak of a solid.

#### Order in a superfluid

The defining property of a superfluid, in contrast, is frictionless flow. It is now widely accepted that superfluidity can be seen as a macroscopic manifestation of quantum particles behaving collectively like a classical complex field [41, 109, 168]. Since the description in terms of a classical field becomes valid in the limit of large occupation numbers, this is intrinsically linked to Bose–Einstein condensation for three-dimensional systems, and the related phenomena in lower dimensions.<sup>5</sup>

As shown by Penrose and Onsager [220], the first-order correlations of a many-body system are, irrespective of interactions, fully described by the spatially-averaged one-body correlation function

$$G^{(1)}(\vec{r}) = \int \langle \hat{\psi}^{\dagger}(\vec{r}')\hat{\psi}^{\dagger}(\vec{r}'+\vec{r})\rangle \,\mathrm{d}^{d}r' \tag{69}$$

computed from the field operators  $\hat{\psi}(\vec{r})$  in position representation, and denoting by  $\langle ... \rangle$  the expectation value for a given many-body state. The momentum density, on the other

<sup>5</sup> For uniform systems in d < 3 dimensions, thermal fluctutations prevent the establishment of a true order parameter [69, 137, 195], but still, power-law correlations in the order parameter field allow to identify a transition (BKT) corresponding to the BEC in three dimensions [24, 25, 41, 99, 128, 158]. See also the appendix, § A.4.4.

hand, is given by  $n(\vec{k}) = \langle \hat{\psi}^{\dagger}(\vec{k}) \hat{\psi}(\vec{k}) \rangle$  with the field operators in momentum representation. A straight-forward calculation<sup>6</sup> then reveals the Fourier relation

$$n(\vec{k}) = \mathscr{F}\{G^{(1)}(\vec{r})\}_{\vec{k}}$$
(70)

which immediately implies that if there is a delta-like peak  $\delta(\vec{k})$  somewhere in the momentum density, such as for a Bose–Einstein condensate, then the one-body density matrix does not go to zero over distance, and vice versa:

$$n(\vec{k}) = \frac{N_0}{V}\delta(\vec{k}) + n_T(\vec{k}) \quad \iff \quad G^{(1)}(\vec{r}) \stackrel{r \to \infty}{\longrightarrow} \frac{N_0}{V} = \text{const.} > 0$$

This corresponds to an infinite correlation length in the system, where each condensate particle must be imagined as essentially delocalised over the whole system [171]. Such a type of order is called off-diagonal long-range order (ODLRO). Equivalently, in the language of symmetry groups, the whole system shares a common condensate phase, which constitutes a breaking of the U(1) gauge symmetry.

Superfluidity in Fermi systems We have seen that superfluidity is intricately connected to a BEC-like phenomenon. But what about superfluid fermionic systems, like superfluid <sup>3</sup>He or the electrons in a superconductor? In this case, the constituent Fermi particles have to pair up and form composite, bosonic objects. These pairs then are allowed to multiply occupy the same state and can undergo a process similar to Bose–Einstein condensation.

# Order in a supersolid

A supersolid, finally, is a state of matter where LRO and ODLRO are simultaneously and spontaneously present. This means that there is a finite fraction of superfluid atoms which is delocalised over a density-modulated volume.

# 4.3 EXPERIMENTS WITH NON-DIPOLAR SYSTEMS

Before the age of ultracold atoms since the mid-1990s, fluid helium has long been the only neutral quantum fluid available to experiments.

The excitation spectrum of superfluid helium exhibits a longitudinal phonon branch at low momentum and a free-particle branch at high momentum. In between those branches, Landau proposed a maxon–roton dispersion, intitially only to improve the agreement between his predictions and experimental data [164, 165, 166]. However, later neutron scattering experiments directly verified the proposed spectral shape [212, 260, 298]. Application of pressure leads to a softening of the roton mode, until the superfluid helium eventually freezes [148, 260]. This phase transition has sparked long speculations about superfluidity in crystalline helium and has for decades been at the centre of the quest for evidence of supersolidity.

Indeed, past experiments have seemed to confirm supersolidity of helium by measurement of non-classical rotational inertia using torsion pendula [19, 151, 152], or by searching for mass transport through helium crystals in thin capillaries [242]. However, it turned out later that these findings could equally be explained by other effects such as isotope impuri-

<sup>6</sup> For greater detail, see the appendix, §A.3.

ties or crystal dislocations [63, 64, 153]. So, whereas it seems to be the established opinion that there is a weak effect of defect-mediated supersolidity in polycrystalline helium,<sup>7</sup> it remains an open question to-date whether monocrystalline (bulk) solid helium actually is a supersolid, or not [19, 41].

Due to the difficulties with solid helium mentioned above, a growing amount of attention has therefore been directed to the field of ultracold atoms, where recently important experimental steps towards the realisation of a supersolid state have been achieved. This is maybe not too surprising, since dilute vapours of ultracold atoms present close to ideal quantum many-body systems that offer two striking advantages over real solids:

- They are essentially free of impurities and imperfections that could obscure subtle signals.
- The remarkable degree of freedom to change not only thermodynamic quantities such as (i) temperature and (ii) density, but also to tune (iii) the geometry of the system as well as (iv) the particle interaction potential.

The first experiments aiming at potential supersolid properties were conducted in the early 2010s in the Esslinger group at ETH Zürich, using <sup>87</sup>Rb atoms interacting with the field of an external optical cavity. In their initial experiment, they observed a spontanous self-organisation of a Bose–Einstein condensate on a pre-imposed lattice structure [21, 202], constituting the breaking of a discrete translational symmetry. Such a 'lattice supersolid' does not yet feature the continuous ground-state degeneracy that characterises a supersolid as originally proposed. With a new experiment, the ETH group later also succeeded in creating a state that breaks a continuous symmetry [173] and hence comes closer to the original concept of a supersolid. Still, the periodicity of their density-modulated states is set externally by a cavity mode, wherefore their states are inherently rigid and, for example, do not support low-momentum phonon excitations. The same is true for an alternative approach followed by the Ketterle group at MIT around the same time, which produced a stripe-phase state via spin-orbit coupling in a <sup>87</sup>Rb BEC [176].

So, while these experiments successfully showed that supersolid properties can be obtained in a quantum gas by a smart coupling to light, there are coherent systems of ultracold atoms where the interaction of atoms alone can give rise to a spontaneous density modulation without the need for external light fields, resembling much more the supersolid envisioned in the early days. This is where dipolar quantum gases enter the stage.

<sup>7</sup> I.e., crystal vacancies delocalised over the lattice, which can Bose condense and move without friction.

Part II

# EXPERIMENTS WITH DIPOLAR QUANTUM GASES

# THE ER-DY EXPERIMENT

In the following chapter, an overview of the Innsbruck Er–Dy apparatus is given, with a special focus on the newly developped and implemented high-resolution imaging setup for our steel 'main' science chamber.

# 5.1 THE CENTRAL APPARATUS

The critical temperatures to reach Bose–Einstein condensation in experiments with dilute metallic vapours are typically in the range of tens to hundreds of nanokelvin. The relevant energy scales for experimental quantum gases are hence many orders of magnitude smaller than the ambient temperature of the surrounding laboratory, around 300 kelvin. To avoid disturbances caused by the room-temperature environment, a quantum gas therefore needs to be well isolated, which is realised in experiments by confining it under ultra-high vacuum (UHV). But how to cool, trap, how to peek and poke at something that may not be touched? This is where lasers and magnetic fields come into play.

The central parts of the Innsbruck Er–Dy experiment, including the UHV apparatus, most laser setups and the magnetic-field coils are laid out in great detail in the thesis of Philipp Ilzhöfer [141]. Additional information on the magnetic-field system can be found in the thesis of Gianmaria Durastante [81]. They have designed and constructed the majority of the central apparatus, wherefore I am much indebted to them. In the following, only a brief overview of this central apparatus<sup>1</sup> and its capabilities is given.

The Er–Dy experiment was designed with the goal of maximum versatility for studies of quantum gases of erbium, of dysprosium, as well as mixtures of both. Typically, adding a second atomic species to a quantum gas experiment more than doubles the complexity of the apparatus, since some components, like laser sources, are required twice, for other components, like shared optics, complex solutions might be required to satisfy the needs of both species. Luckily, due to the similarity of many atomic properties of the lanthanoid elements erbium and dysprosium, it was possible to engineer a vacuum apparatus that, at first glance, bears much of the simplicity of a single-species experiment. The apparatus is divided in three modules separated by gate valves, which are closed manually, e.g., when erbium and dysprosium source material in the oven is used up and needs to be replenished.

In the first module, evacuated to the low  $10^{-10}$  millibar level, erbium and dysprosium are evaporated in a single, commercial dual-filament effusion cell ('oven' in Fig. 6), where the source granulate<sup>2</sup> is stored and heated to typically 1100 °C and 1200 °C in the 'bulk zone' and 'hot lip' region of a tantalum crucible, respectively (for details, see Ref. [141]).

A versatile mixture experiment

Module 1: Atomic beam source

<sup>1</sup> This is, in particular, the modules of the apparatus needed for experiments conducted inside what is colloquially termed the 'main chamber' see Fig. 6. For the lattice microscope, whose design forms an integral part of this present thesis, the reader is referred to § 10.

<sup>2</sup> In the past, we have used a granulated alloy of erbium and dysprosium in ratio 1 : 2 [141]. Since 2018, we have switched to pure granulates of erbium and dysprosium, which allow to selectively adapt the ratio between the two and their respective fluxes in the atomic beam.



FIGURE 6: A sketch of the central vacuum components of the Er–Dy experiment. Atoms are evaporated in the oven on the right, travel leftward, are collimated via transverse cooling, slowed in the Zeeman slower and finally captured in the MOT. Figure modified from Ref. [142].

The evaporated atoms pass through a series of apertures and form an atomic beam, which is collimated by a 2D optical molasses stage ('transverse cooling') using the broad atomic transitions (401 nm for erbium, 421 nm for dysprosium, with line widths  $\Gamma_{401}/2\pi \approx 29.4$  MHz and  $\Gamma_{421}/2\pi \approx 32.2$  MHz).

After passage of the first gate valve, the atomic beam enters the second module, where a mechanical shutter allows to cut the atom flux (typically after each MOT loading step). Additionally, this section acts as the middle stage of a differential pumping scheme. At a pressure on the  $10^{-11}$  millibar level, it separates the lower-vacuum oven region from the higher-vacuum main chamber.

Through the second gate valve, the atomic beam enters the main chamber module. The atomic beam is decelerated using a Zeeman slower in decreasing–zero–increasing field<sup>3</sup> configuration, using the broad blue transitions of erbium and dysprosium. Subsequently, the atomic beam enters the main chamber. Here, at a pressure on the low  $10^{-11}$  millibar level, the atoms are finally captured in a magneto-optical trap (MOT) using the 583 nm and 626 nm intercombination transitions of erbium and dysprosium, respectively, with line widths  $\Gamma_{583}/2\pi \approx 186$  kHz and  $\Gamma_{626}/2\pi \approx 135$  kHz. An electro-optic modulator (EOM) is used to modulate strong sidebands onto the carrier MOT light, which are spaced by around 100 kHz and address atoms of different velocity classes from the atomic beam, thus increasing the MOT capture efficiency [142].

Five-beam configuration

Module 2: Atomic beam shutter

> Module 3: Main chamber

Narrow-line MOT

The narrow-line character of the intercombination transitions brings about two major advantages. First, in combination with the high atomic mass of erbium and dysprosium, it allows to operate this MOT in a unique five-beam configuration, as first demonstrated by our group in Ref. [142]. This is very advantageous, since it automatically guarantees a spin-polarised sample in the lowest Zeeman sublevel, while at the same time it frees

<sup>3</sup> Colloquially (but unfortunately not strictly correct from a physical perspective) such a design is often termed a 'spin flip' configuration.

the optical access from above (see Fig. 6 on the facing page), allowing the close placement of high-resolution imaging optics (described in the next section, § 5.2). The erbium and dysprosium MOTs can be displaced vertically with respect to each other by independent adjustment of the individual MOT detunings. Therefore, a negative cross-species influence of the MOTs in mixture operation can be efficiently mitigated. Additionally, the narrow-line character of the intercombination transitions leads to low Doppler temperature limits below 5  $\mu$ K. After typical MOT loading times of around 5 s, we apply a compressed-MOT step by decreasing (i) the magnetic field gradient, (ii) the MOT light detuning and (iii) power, yielding both a lower temperature (around 10  $\mu$ K for both species [142]) and a spatial compression of the sample, facilitating the direct loading into our optical dipole trap (ODT). In our compressed MOT, we currently capture about (5 ± 2) × 10<sup>7</sup> atoms.

The ODT consists of two near-infrared laser beams<sup>4</sup> crossed in the horizontal (*xy*) plane<sup>5</sup> under 45°. The beam perpendicular to the Zeeman slower (along *y*) has a waist of  $w_{x,z} \approx 18 \,\mu\text{m}$ , with the focus position in the Fourier plane of an acousto-optic deflector (AOD). Fast scanning of the centre frequency of the AOD allows to paint time-averaged potentials<sup>6</sup> along *x* (marked by an overline) and thus to tune the geometry of the harmonic trap aspect ratio  $\overline{w}_x : w_z$  between roughly 1 : 1 and 10 : 1. The diagonal, 'static' ODT beam has a waist of about 60  $\mu\text{m}$ .<sup>7</sup>

Once the atoms are loaded into the ODT, the MOT light is switched off and the homogeneous magnetic bias field along the vertical direction (z) is set to a value where the atomic scattering lengths are such that two-body collision rates are sufficiently high for an efficient evaporative cooling, but losses due to three-body collisions are not too severe. This magnetic bias field additionally ensures the preservation of spin polarisation of the trapped atoms. Forced evaporation, down to quantum degeneracy, is then performed by decreasing the laser power near-exponentially, approximated by piecewise linear ramp sections, on a timescale of a few seconds.

# Degenerate samples and mixtures

Natural erbium and dysprosium offer five bosonic ( ${}^{166}\text{Er}$ ,  ${}^{168}\text{Er}$ ,  ${}^{170}\text{Er}$ ,  ${}^{162}\text{Dy}$ ,  ${}^{164}\text{Dy}$ ) and three fermionic isotopes ( ${}^{167}\text{Er}$ ,  ${}^{161}\text{Dy}$ ,  ${}^{163}\text{Dy}$ ) with abundances above ten percent. Of those, samples of the isotopes marked by the underline<sup>8</sup> have been brought to quantum degeneracy on the Innsbruck Er–Dy machine to-date [283].

When producing heteronuclear mixtures, the slightly lower ODT depth experienced by erbium leads to preferential evaporation and a significant sympathetic cooling effect Optical dipole trap

Evaporative cooling

Abundant isotopes

Degenerate mixtures

<sup>4</sup> At 1064 nm, delivered by a Mephisto MOPA (Coherent, Inc.) with 55 W nominal output power, split into two branches and mode-cleaned in parallel through two photonic crystal fibres (Alphanov).

<sup>5</sup> The laboratory coordinate system, centred at the crossing point of the ODT beams, is such that the atomic beam propagates into the positive *x*-direction and *z* is counter-directed to gravity; cf. Fig. 6.

<sup>6</sup> Details about the scanning ODTs implemented in the Ferlaino labs can be found in the Master theses of Simon Baier [15] and Claudia Politi [237].

<sup>7</sup> In March 2021, we have added a second diagonal 'static' ODT beam with roughly equal specifications as the first one, which it crosses at right angle. In contrast to the former two-beam arrangement at 45°, the crossing of all three beams now allows to create traps which are isotropically round in the horizontal *xy*-plane. The vertical trap frequency along the *z* direction can additionally be varied via the 'scanning' beam.

<sup>8 &</sup>lt;sup>167</sup>Er suffers from light-induced losses in a 1064-nm ODT, at least above the ultracold-temperature regime [2], and hence requires an intermediate ODT stage of different wavelength (such as 532 nm or 1550 nm). <sup>163</sup>Dy, due to its inverted hyperfine structure, would most probably require a dedicated optical pumping scheme [283].

on dysprosium [283]. In mixture operation, so far five heteronuclear double-BECs (<sup>166</sup>Er–<sup>164</sup>Dy, <sup>168</sup>Er–<sup>162</sup>Dy, <sup>168</sup>Er–<sup>164</sup>Dy, <sup>170</sup>Er–<sup>162</sup>Dy, <sup>170</sup>Er–<sup>164</sup>Dy) and one heteronuclear degenerate Bose–Fermi mixture (<sup>168</sup>Er–<sup>161</sup>Dy) have been demonstrated [ibid.].

Interspecies Feshbach resonances We have studied the interspecies Feshbach spectra of heteronuclear mixtures of erbium and dysprosium in detail and were able to identify, among many narrow features, two resonances of a width greater than 1G (one for  $^{166}\text{Er}-^{164}\text{Dy}$ , one for  $^{168}\text{Er}-^{164}\text{Dy}$ ), and an on-resonance loss rate suggestive of the universal scaling relation associated with broad resonances. Further, we have observed an increased density of Feshbach resonances in the mixture compared to the single-species case. These results have been presented in Refs [81, 82].

# 5.2 A HIGH-RESOLUTION IMAGING SYSTEM FOR BULK DIPOLAR QUANTUM GASES

The majority of quantum gas experiment relies on optical imaging techniques to read out the density distribution of a sample.<sup>9</sup> The most established approach for optical imaging of quantum gases is absorption imaging (see § 5.2.3), which works well for samples that are released from the trap and expand during a few to tens of milliseconds time of flight (TOF). During expansion, with increasing TOF, the momentum distribution of the atoms is mapped onto position space and can be read out from the image.<sup>10</sup> Many groundbreaking discoveries have been made using the TOF technique, including the demonstration of the first Bose–Einstein condensates [5, 74], or the superfluid to Mott insulator transition [115].

The direct information about the in-trap density distribution, however, gets lost during the TOF expansion.

To access this information, much interest has been directed towards imaging quantum gases in trap. Alas, this is not a trivial task since trapped quantum gases are small objects on the order of a few micrometres, and resolving their substructure requires imaging optics with a sufficient numerical aperture (NA). This can present a technical challenge for cold atom experiments, where typically long working distances are required due to the confinement in UHV. Additionally, in-situ samples are typically too dense for standard absorption imaging.

Top-down approach With the advancement of technology, however, much progress has recently been made in imaging quantum gases in situ. The experimental strategies for this can be divided into a top-down and a bottom-up approach. The top-down approach consists of imaging many-body quantum systems with higher and higher precision. Iconic examples in this context include partial-transfer absorption imaging of dense BECs for vortex studies [200, 239, 240], in-situ imaging of two-dimensional systems [68, 70, 252, 287, 305], the discovery of quantum droplet states of dipolar gases [91, 145, 259], and – the ultimate limit – quantum gas microscopes, capable of distinguishing (and typically also of manipulating) individual atoms on an optical lattice [18, 267]. The bottom-up approach, in contrast, relies on assembling many-body quantum systems one by one from individual particles. Examples of this approach include the single-atom imaging of few-fermion systems in a

Bottom-up approach

<sup>9</sup> The BECs of metastable helium present an exemption, since they can imaged electronically using microchannel plates [221, 248].

<sup>10</sup> Non-ballistic ('hydrodynamic') expansion can influence the mapping, e.g., for strongly interacting systems (see Refs [98, 284], e.g.), but does not present a fundamental obstacle for absorption imaging.

(single) optical tweezer [23, 26, 138], or arrays of multiple optical tweezers that contain single (often Rydberg) atoms (for recent reviews see, e. g., Refs [1, 51]).

Also with the Er–Dy experiment we have taken steps towards high-resolution imaging, following above's top-down approach.<sup>11</sup> The development of a dipolar quantum gas microscope for erbium and dysprosium atoms on optical lattices is described in § 10 later in this thesis. The design and implementation of a high-resolution optical system for in-situ imaging of bulk quantum gases of erbium and dysprosium and their exotic phases such as quantum droplets and dipolar supersolids is discussed in the following. Many features of the imaging techniques applied by us have already been presented elsewhere in similar form (Refs [8, 45, 46, 145, 293], and many others), nevertheless, important details are recapitulated here for context.<sup>12</sup> For a deeper theoretical background on how optical images form, the reader is referred to the appendix, § B.

## 5.2.1 Light field modifications through a medium

In the following, let us briefly review the basic concepts of how light fields are modified by the presence of an optical medium.



FIGURE 7: Schematic of the elements important for the image recording. A = probe beam of defined polarisation, B = atomic sample, C = vacuum window, D = imaging objective, E = back focus, F = iris diaphragm, G = imaging lens, H = optional linear polariser, J = camera.

A medium is characterised by the complex index of refraction<sup>13</sup>  $n_c = n_r + i\kappa = \sqrt{\varepsilon_r \mu_r}$ , which quantifies the reduction of the speed of light in the medium compared to vacuum,  $c' = c/n_c$ . Accordingly, the vacuum wave vector  $k = \omega/c$  is modified to  $k' = \omega/c'$  inside the medium. If we consider light propagating along  $\vec{e}_z$ , the field and intensity inside a dielectric can thus be written as

$$E(z,t) = E_0 e^{i(k'z-\omega t)} = E_0 e^{i(kz-\omega t)} e^{i\beta} \quad \text{and}$$
(71)

$$I(z,t) = |E(z,t)|^2 = I_0 e^{-\epsilon},$$
(72)

<sup>11</sup> In our group's T-REQS laboratory, the bottom-up approach is followed in the development of a tweezer array for erbium Rydberg atoms.

<sup>12</sup> Due to comparable requirements, our high-resolution vertical imaging setup shares much similarity with the dysprosium experiment in Stuttgart. We have benefitted in many respects from the experiences of the Stuttgart group, whose setup is well described in the thesis of Matthias Wenzel [293].

<sup>13</sup>  $n_r$  is the familiar, real part of the refractive index,  $\kappa(\omega)$  is the extinction coefficient of the Beer–Lambert law  $(dI/dz = -\kappa I)$ ,  $\varepsilon_r$  is the relative permittivity,  $\mu_r$  the relative permeability and *c* the vacuum speed of light.

with a complex phase

$$\beta = \varphi + \frac{i}{2}\epsilon, \tag{73}$$

whose real part  $\varphi = (n_r - 1)kz$  is called the dispersive phase, and whose coefficient of the imaginary part,  $\epsilon = 2\kappa kz$ , is called the optical depth (OD)<sup>14</sup> of the medium [46].

In the case  $|n_c| \sim 1$ , as in a dilute vapour, the *macroscopic* quantities  $n_r$  and  $\kappa$  are directly connected to the real and imaginary part of the *microscopic* polarisability<sup>15</sup>  $\alpha$  by

$$\beta(\omega, z) = \frac{\omega}{2\varepsilon_0 c} \bar{n}_z \alpha(\omega), \tag{74}$$

where

$$\bar{n}_{z}(x,y) = \int_{0}^{z} n(x,y,z') dz'$$
(75)

is the column density of the dielectric medium, integrated up to the observation plane at *z*.

At low light intensity,  $\alpha$  can be derived from the Lorentz model of the atom as a classical oscillator [118], however, a semiclassical<sup>16</sup> correction is needed at high intensity when the atomic ground state becomes increasingly depleted, leading to a saturation of the scattering rate. Application of the rotating-wave approximation (RWA) for the polarisability  $\alpha(\omega)$  and inserting the result into Eq. 74 yields, at position  $\vec{r} = (x, y, z)^{\mathsf{T}}$ ,

$$\varphi(\vec{r},\Delta) = -\frac{\sigma_0 \bar{n}_z(x,y)\Gamma\Delta}{4\Delta^2 + \Gamma^2(1+I/I_s)} \qquad \propto \Re\{\alpha(\omega)\} \qquad \text{and} \tag{76}$$

$$\epsilon(\vec{r},\Delta) = +\frac{\sigma_0 \bar{n}_z(x,y)\Gamma^2}{4\Delta^2 + \Gamma^2(1+I/I_{\rm s})} \qquad \propto \Im\{\alpha(\omega)\},\tag{77}$$

where  $\Gamma$  is the spontaneous excited-state decay rate,  $\Delta = \omega - \omega_0$  is the detuning from the atomic resonance,  $\sigma_0 = 6\pi c^2/\omega_0^2$  is the scattering cross section on resonance, and  $I_s = \hbar\Gamma\omega_0^3/12\pi c^2$  is the saturation intensity on resonance [46, 102].

In Fig. 8, both  $\epsilon(\Delta)$  and  $\varphi(\Delta)$  are plotted at low and at saturation light intensity, respectively. Three important aspects become apparent:

- On resonance, the optical depth  $\epsilon$  has its maximum, whereas the dispersive phase  $\varphi$  vanishes.
- Away from resonance,  $\epsilon$  approaches zero as  $(\Gamma/\Delta)^2$ , whereas  $|\varphi|$  drops much more slowly, as  $\Gamma/\Delta$ .
- High light intensity reduces both the optical depth and dispersive phase shift. Since this increases the FWHM, such a line is called power-broadened.

<sup>14</sup> The terms *optical depth, absorbance* and *extinction* denominating the quantity  $\ln(I/I_0)$  are often used interchangeably in literature, however, not always with consistent definitions, especially concerning the choice of natural or decadic logarithm.

<sup>15</sup> Throughout this work, the definition of Grimm et al. [118] is chosen,  $\vec{p} = \alpha \vec{E}$ .

<sup>16</sup> I.e., quantum-mechanical modelling of the atom as a two-level system and classical treatment of the light field, see, e.g., Ref. [102].



FIGURE 8: The optical depth  $\epsilon$  and dispersive phase  $\varphi$  for light interacting with a gas of unit column density ( $\bar{n}_z \equiv 1$ ), at low and at saturation intensity.

Equations 76–77 allow to calculate the sample column density  $\bar{n}_z(x, y)$  if either  $\epsilon$  or  $\varphi$  is known. As will be discussed in §§ 5.2.3–5.2.4, in an experiment  $\epsilon$  can be determined by absorption imaging, whereas  $\varphi$  can be determined by phase-contrast imaging. For many situations in cold-atom experiments a theoretical model for the 3D density distribution (either in trap or after TOF) is available, allowing to reconstruct the 3D density from the 2D column densities.

# 5.2.2 Birefringence of polarised atomic vapours

In the laboratory one is typically concerned with atoms oriented in an external (usually magnetic) field. For the following discussion, we will assume the atoms at the coordinate origin and call the quantisation axis the *z*-direction. The response of an atom to an oncoming light field then depends on the relative orientation of the light propagation direction  $(\vec{k})$  and the quantisation axis, allowing to distinguish two limit cases [102]:

- $\vec{k} \perp \vec{e}_z$  atoms absorb/emit linearly polarised light
- $\vec{k} \parallel \vec{e}_z$  atoms absorb/emit circularly polarised light

The latter setting is used for imaging erbium and dysprosium in our experiments, since it allows to cycle on a quasi-closed transition (see Fig. 9 on the next page).

As described earlier (§ 5.1), the five-beam MOT automatically provides atomic samples fully spin-polarised in the lowest Zeeman sublevel of the electronic ground state. From this state, light copropagating with the quantisation axis can excite  $\sigma^{\pm}$  transitions to the respective  ${}^{1}P_{1}$  state (Fig. 9). At typical magnetic fields, the splitting between adjacent Zeeman sublevels (on the order of 1 MHz/G [16, 104]) is much less than the linewidth of the transition.

Denoting angular-momentum eigenstates by  $|J, m_J\rangle$ , the probabilities of a single  $\sigma^{\pm}$  excitation,

$$|J, m_J\rangle \xrightarrow{\sigma^+} |J+1, m_J+1\rangle$$
 or  $|J, m_J\rangle \xrightarrow{\sigma^-} |J+1, m_J-1\rangle$ ,



(a) 401-nm  $\sigma^{\pm}$  light driving the erbium  $|6, -6\rangle \rightarrow |7, -6 \pm 1\rangle$  transition with the corresponding Clebsch-Gordan coefficients.

(b) 421-nm  $\sigma^{\pm}$  light driving the dysprosium  $|8, -8\rangle \rightarrow |9, -8 \pm 1\rangle$  transition with the corresponding Clebsch-Gordan coefficients.

FIGURE 9: The strongest transitions of erbium and dysprosium for  $\vec{k} \uparrow \uparrow \vec{B}$  from the lowest Zeeman sublevel of the ground states to the respective  ${}^{1}P_{1}$  states. The blurred arrowheads indicate that the energy splitting is less than the linewidth.

are given by the squares of the corresponding Clebsch–Gordan coefficients [196],

$$C_{\pm} = \langle J, m_I; 1, \pm 1 | J + 1, m_I \pm 1 \rangle.$$
(78)

This means that for erbium and dysprosium a  $\sigma^+$ -interaction event of an atom in the lowest Zeeman sublevel is suppressed by factors of  $C_+^2 = 91$  and  $C_+^2 = 153$  compared to  $\sigma^-$ , respectively, where  $C_-^2 = 1$  (Fig. 9).

# 5.2.3 Absorption imaging

Absorption imaging relies on the imaginary part of the complex phase  $\beta$ . As seen in Eq. 72, a finite optical depth leads to a reduction of light intensity on the image sensor; the sample 'casts a shadow'.

We use absorption imaging to extract quantitative information on the density distribution information of samples of erbium and dysprosium atoms after time-of-flight (TOF) expansion. To this end, we employ probe beams on resonance<sup>17</sup> with the broad blue transitions at 401 nm and 421 nm, respectively. The short wavelength allows a high imaging resolution according to the Abbé limit, the large linewidth allows for high scattering rates and hence for short imaging pulses. The probe beam polarisation is chosen such that it excites a  $\sigma^-$  transition between two stretched Zeeman substates, as illustrated in Fig. 9.

Experimentally, pictures are recorded with an image sensor<sup>18</sup>, which, exposed to a true intensity I, will typically give out a linear response

$$I' = aI + b \tag{79}$$

with the responsivity a and an offset b. To relate the measured intensity I' to the sample density it would be necessary to know the image sensor characteristics a and b with high

<sup>17</sup> As seen before, on resonance the dispersive phase vanishes (Eq. 76) and the signal is purely absorptive.

<sup>18</sup> Established sensor types for scientific experiments include charge-coupled device (CCD) and scientific complementary metal-oxide semiconductor (SCMOS) sensors.
accuracy. It is scientific standard to circumvent this requirement by taking three pictures per sample, one with atoms  $(I'_1)$ , one without atoms  $(I'_2)$ , and a dark picture without probe light  $(I'_3)$ , such that [293]

$$I'_1 = aI_0 e^{-\epsilon} + b, \qquad I'_2 = aI_0 + b, \qquad I'_3 = b.$$
 (80)

These three pictures can be combined to an image of the optical depth by

$$\epsilon = -\ln\left(\frac{l_1' - l_3'}{l_2' - l_3'}\right),\tag{81}$$

which is independent of the parameters a and b. On resonance, the absorption signal is maximal (Fig. 8) and related to the column density by

$$\epsilon_0(x,y) \equiv \epsilon(x,y;0) = \frac{\sigma_0 \bar{n}_z(x,y)}{1 + I/I_s}.$$
(82)

The dynamic range of image sensors sets a technical limit to the detection range of the OD for standard, low-intensity ( $I_0 \ll I_s$ ) absorption imaging, since signals for  $\epsilon \gtrsim 3$  are often below the noise level, resulting in 'saturated' OD images. As mentioned, saturation can be mitigated either by releasing the cloud into time-of-flight expansion, during which the density decreases, or, alternatively, by using high-intensity probe beams ( $I_0 \gg I_s$ ) [245], which reduces the effective OD via Eq. 77 (cf. Fig. 8 on page 45). Then, however, caution is needed, since high intensities may lead to a strong heating of the sample.

An alternative approach that does not suffer from these limitations is phase-contrast imaging, as described in the following section.

#### 5.2.4 Phase-contrast imaging

Phase contrast imaging relies on the real part of the complex  $\beta$  (Eq. 73). At detunings several  $\Gamma$  away from resonance,  $\beta$  becomes small and effectively purely dispersive. Since photo detectors are only sensitive to intensity, not fields, the field variations caused by dispersive phase shifts acquired in the sample first have to be converted into intensity variations. This is achieved using interference.

For a birefringent medium, such as a polarised atomic cloud, there is a particularly simple way to implement phase contrast imaging. For this method, linearly polarised light is sent through the sample along the direction of the quantisation field (*z*). In this case, the incident light, whose initial linear polarisation direction we will call *x* (without loss of generality) is best described in terms of the Jones basis of right (+) and left (-) circular polarisation,

$$\vec{E}_0 = E_0 \vec{e}_x = \frac{1}{\sqrt{2}} E_0 (\vec{e}_+ + \vec{e}_-)$$
 with  $\vec{e}_{\pm} = \frac{1}{\sqrt{2}} (\vec{e}_x \pm i \vec{e}_y)$  (83)

which induce atomic  $\sigma^+$  and  $\sigma^-$  transitions, respectively [253, 293]. Due to the difference in Clebsch–Gordan coefficients, the  $\sigma^-$  component is much more likely to interact with the

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atoms and gets retarded, turning the initial linear polarisation into an elliptic polarisation of the scattered light field  $\vec{E}_s$ :

$$\vec{E}_{\rm s} = \frac{1}{\sqrt{2}} E_0 \left( \mathrm{e}^{\mathrm{i}\beta_+} \vec{e}_+ + \mathrm{e}^{\mathrm{i}\beta_-} \vec{e}_- \right) \stackrel{(*)}{\approx} \frac{1}{\sqrt{2}} E_0 \left( \vec{e}_+ + \mathrm{e}^{\mathrm{i}\varphi} \vec{e}_- \right)$$
(84)

with 
$$\beta_{\pm} = \frac{C_{\pm}^2}{C_{\pm}^2 + C_{-}^2} \beta(\Delta_{\pm}).$$
 (85)

The approximation (\*) is well justified in our case where the difference in Clebsch–Gordan coefficients ( $C_{\pm}$ ) is large, the splitting between Zeeman sublevels is negligible compared to the linewidth, and we are sufficiently detuned from resonance, i. e.  $|\Delta_{+} - \Delta_{-}| \ll \Gamma \ll |\Delta_{\pm}|$ , where  $\Delta_{\pm}$  is the respective frequency detuning from the  $\sigma^{\pm}$  transition.

If the scattered light field (85) is passed through a linear polariser, rotated by an angle  $\vartheta$  with respect to x, the two polarisation components ( $\pm$ ) interfere and the resultant field  $\vec{E}_{p}$  reads, in Jones formalism:<sup>19</sup>

$$\vec{E}_{p} = R(\vartheta) \cdot A_{x} \cdot R^{-1}(\vartheta) \cdot \vec{E}_{s}$$
(86)

$$= \begin{pmatrix} C & -S \\ S & C \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} C & S \\ -S & C \end{pmatrix} \cdot \vec{E}_{s} = \begin{pmatrix} C^{2} & CS \\ CS & S^{2} \end{pmatrix} \cdot \vec{E}_{s}$$
(87)

The intensity after the polariser then becomes, after some algebra,

$$\frac{I_{\rm p}}{I_0} = \frac{|\vec{E}_{\rm p}|^2}{|\vec{E}_0|^2} = \cos^2(\frac{\varphi}{2} - \vartheta) = \cos^2\vartheta + \frac{\varphi}{2}\sin(2\vartheta) - \frac{\varphi^2}{4}\cos(2\vartheta) + \mathcal{O}(\varphi^3)$$
(88)

when expanding for small angles  $\varphi$ . For polariser orientations  $\vartheta$  at 45° and 90°, this yields:

$$\frac{I_{\rm p}}{I_0} \approx \begin{cases} \frac{1}{2}(1+\varphi) \text{ for } \vartheta = 45^{\circ} \\ \frac{1}{4}\varphi^2 \quad \text{for } \vartheta = 90^{\circ} \end{cases}$$
(89)

Dark-field imaging The case  $\vartheta = 90^{\circ}$  is called dark-field imaging, since light that has not gained a phase shift from interaction with the sample is blocked by the polariser and does not reach the image sensor. It has been used, e.g., for nondestructive in-trap imaging of sodium BECs at MIT early on [8], however, the signal is quadratic in  $\varphi$  and, since  $\phi \ll 1$ , small.<sup>20</sup>

At  $\vartheta = 45^\circ$ , the signal is linear in  $\varphi$  and hence easier to detect. This case constitutes Faraday phase contrast imaging in the narrower sense. It has been first employed at Rice University to image small lithium BECs directly in their magnetic trap [45, 47], where due to attractive interactions stable condensates are limited to around a thousand atoms, and later in the context of dipolar quantum gases in Stuttgart [145, 293].

Faraday imaging

<sup>19</sup> *R* is the rotation matrix,  $A_x$  is a polariser along x,  $C \equiv \cos \vartheta$  and  $S \equiv \sin \vartheta$ .

<sup>20</sup> The implementation in the MIT experiment was in fact slightly different, without polariser but by simply blocking the unrefracted light in the back focus of the objective (element *E* in Fig. 7 on page 43). The effect is, however, the same.

We use the latter method ( $\theta = 45^{\circ}$ ) at high detuning  $\Delta \sim 18 \Gamma$  for in-situ imaging of our dipolar quantum gases.<sup>21</sup> At such a high detuning, Eq. 76 simplifies to

$$\varphi(\vec{r},\Delta) \approx -\frac{1}{4}\sigma_0 \bar{n}_z(x,y) \frac{\Gamma}{\Delta},\tag{90}$$

where we have exploited that at  $\Delta \gg \Gamma$  typically  $I \ll I_s$  is easily fulfilled.

Similar to absorption imaging, by combination of three consecutive experimental pictures the sensor characteristics can be eliminated from the sample image. Starting from Eq. 89 for  $\vartheta = 45^\circ$ , the images with atoms  $(I'_1)$ , without atoms  $(I'_2)$  and without probe beam  $(I'_3)$  read

$$I'_{1} = \frac{a}{2}(1+\varphi)I_{0} + b, \qquad I'_{2} = \frac{a}{2}I_{0} + b, \qquad I'_{3} = b.$$
(91)

From this we calculate  $\phi$  and, via Eqs. 76–77, the off-resonant (low-intensity) OD as [293]

$$\varphi(\Delta) = \frac{I'_1 - I'_2}{I'_2 - I'_3} \quad \text{and} \quad \varepsilon(\Delta) = -\frac{\Gamma}{\Delta}\varphi(\Delta).$$
(92)

The corresponding OD which would be measured for a probe beam on resonance,  $\epsilon_0 \equiv$  $\epsilon(\Delta=0)$ , can be calculated by comparison of Eqs 82 and 90, giving

$$\epsilon_0 = -4\frac{\Delta}{\Gamma}\varphi(\Delta). \tag{93}$$

From  $\epsilon_0$  it is straight forward to calculate the column density of the sample.

#### 5.2.5 Experimental implementation

As depicted schematically in Fig. 7, our vertical imaging system consists of an imaging ob-Optical layout jective, an imaging lens, and a camera. The objective itself is a custom, infinite-conjugate system<sup>22</sup> and achromatic for the three wavelengths 401 nm, 421 nm and 633 nm. It consists of two singlet and one cemented doublet lens, each with a broad-band (and broadangle) dielectric anti-reflection coating from 375 to 650 nm.<sup>23</sup> The objective specifications Coating are listed in Table 1.

For the imaging lens, we have two options to pick from, depending on the requirements of the current research project: a commercial broadband achromat<sup>24</sup> with focal length f' = 1.00 m, and a custom air-spaced doublet lens<sup>25</sup> with f' = 1.75 m.

The sizes of an object (h) and its image (h') are related by the lateral magnification Magnification M of the optical system (cf. Fig. 33 on page XVI). In our case where both are at infinite

<sup>21</sup> For erbium and dysprosium at low field  $\Delta_+ \approx \Delta_-$ , so we may simply write  $\Delta_{\pm} \equiv \Delta$ .

<sup>22</sup> Special Optics, Inc., NJ/USA.

<sup>23</sup> It is crucial that the MOT wavelengths are not reflected by the objective. When aligned, its concave front surface can lead to a focussed reflexion of the vertical MOT beams onto the MOT and disturb it severely (cf. Fig. 11a). 24 ACT508-1000-A, Thorlabs, Inc., NJ/USA

<sup>25</sup> Lens-Optics GmbH, Germany

| QUANTITY                               | DESIGN VALUE      | MEASURED                                |
|----------------------------------------|-------------------|-----------------------------------------|
| working distance                       | 55 mm             |                                         |
| effective focal length                 | 65 mm             |                                         |
| magnification                          | ×15.4 or<br>×27.0 | $\times 15.3(1)$ or<br>$\times 27.3(2)$ |
| max. NA                                | 0.45              |                                         |
| resolution at 401 nm <sup>a</sup>      | 0.54 µm           | 0.66(1) µm                              |
| resolution at 421 nm <sup>a</sup>      | 0.57 µm           | 0.71(1) µm                              |
| Ø diffraction-limited FOV <sup>a</sup> | 150 µm            | $> 130\mu m$                            |

TABLE 1: Optical specifications of the vertical imaging objective. Values in parentheses give the statistical standard uncertainty of the last digit.

<sup>a</sup> Evaluated at full NA.

conjugation (IC), *M* is simply determined by the ratio of the effective focal lengths of objective (*f*) and imaging lens (f'),<sup>26</sup>

$$\frac{h'}{h} \equiv M \stackrel{\text{(IC)}}{=} \frac{f'}{f}.$$
(94)

Image sampling

For in-situ imaging of quantum gases one generally aims for magnifications  $M \gg 1$  to guarantee a sufficient sampling of the image by the image sensor, which typically has a pixel size of a few microns.<sup>27</sup> An image is limited by the physical resolution, rather than sampling, if the Nyquist-Shannon criterion is fulfilled [192, 193, 266]:

$$\nu_{\rm s} > \nu_{\rm b} \qquad \Longleftrightarrow \qquad \frac{1}{d_{\rm px}} > \frac{4\,{\rm NA}}{|M|\lambda}$$
(95)

In Eq. 95,  $v_s$  is the sampling rate of the image related to the sensor pixel size  $d_{px}$ ,  $v_b$  is the bandwidth of the imaging system itself, whose cut-off depends on the object-space NA, the magnification M and the imaging wavelength  $\lambda$ .

Of course, there is a practical limit to the magnification. The photon count rate per pixel scales as  $\propto M^{-2}$ , therefore at some point the signal-to-noise ratio (SNR) will increase. For typical magnifications, saturated atomic photon emission, and the noise levels of modern scientific cameras, however, this is usually not the limiting factor.

Our 1-m achromat gives around  $\times 15$  magnification, which is below the Nyquist–Shannon criterion, but still good enough to adequately image arrays of quantum droplets, as will be detailed later. This low magnification, however, allows for a larger field of view, which is necessary, e.g., for sample thermometry, where we rely on the density profile of the background thermal fraction after TOF expansion. If in the future it becomes necessary to study structures on smaller scales, such as density fluctuations *inside* quantum

<sup>26</sup> In the general case this needs not be true and *M* has to calculated from the ratio of object- and image space numerical aperture (NA) [269, §9.7], |M| = NA/NA'; cf. Fig. 33 in the appendix, §B.2.2.

<sup>27</sup> In our case, an Andor Neo 5.5 SCMOS camera with  $d_{px} = 6.5 \,\mu\text{m}$  pixel size.

droplets, we will switch to the 1.75-m achromat, giving  $\times$ 27 magnification and meeting the Nyquist–Shannon criterion up to NA  $\approx$  0.42.

As a first simple offline test of the newly designed objective, imaging of a USAF-1951 resolution test chart<sup>28</sup> revealed an upper bound for the resolution of about  $1.5 \,\mu$ m. For a finer characterisation, more sophisticated methods are needed.

In general, the performance of an imaging system is fully described once we know

- 1. the object-space numerical aperture (NA), which determines the diffraction limit, and
- 2. the aberrations of the system, which determine how much the resolution of the actual system is reduced compared to an ideal, aberration-free system.

While the NA of the system is typically known or relatively easy to estimate, the aberrations have to be measured or, at least, simulated numerically. For any imaging system, the aberrations can be thought of as the wavefront error W(x, y) compared to a perfectly spherical wave in the exit pupil [112].<sup>29</sup> A commonly used definition for a diffraction-limited system is the Maréchal criterion [191], which corresponds to a root mean square wavefront error

$$W_{\rm RMS} = \sqrt{\langle W^2(x,y) \rangle - \langle W(x,y) \rangle^2} \le \frac{\lambda}{14}.$$
(96)

The wavefront error W(x, y) can be simulated if a numerical model of the imaging system is at hand (cf. § 11.2), and also be measured in the laboratory. One measurement approach is optical interferometry, as has been done for the microscope experiments at Harvard [219] and MIT [241] using Fizeau interferometers. A different approach is phase retrieval via nonlinear optimisation, as has been demonstrated in Innsbruck [193] following a proposal by Brady & Fienup [48]. For this method, measurements of the light intensity distribution in different planes around the focus are sufficient for reconstructing the wavefront in the exit pupil, without a need for interferometry.

A complementary option to characterise the aberrations present in an optical system consists in measuring its amplitude point-spread function (PSF), which is the Fraunhofer diffraction pattern of the exit pupil and therefore directly connected to W(x, y) (see the appendix, §B.3.2). If the input of an imaging system is an ideal point source, the produced image is directly equal to the system's PSF. Therefore, if a good approximation of a point source is available, it is straight forward to measure the PSF in the laboratory.

Common choices for small sources include pinholes and nanoparticles. However, mechanical pinholes are typically available with diameters down to 1 µm and therefore not small enough for testing high-resolution systems. In contrast, nanoparticles, for example of gold or TiO<sub>2</sub>, have small diameters down to  $\sim 20$  nm, but the scattered light has typically very low intensity and an unknown polarisation. We therefore chose to follow a less-known approach, originally demonstrated by the Meschede group in Bonn [49, 155, 247], imaging the tip of a SNOM fibre<sup>30</sup> which has a nominal aperture of 50 to 100 nm and an easily sufficient maximum output power of 400 µW. Resolution test of the objective

Wavefront error

Point-spread function

<sup>28</sup> Edmund Optics, Inc., NJ/USA

<sup>29</sup> See the appendix, § B, for background theory on aberrations, and image formation in general.

<sup>30</sup> Optical fibres for scanning near-field optical microscopy (SNOM); model MF001 from Tipsnano OÜ, Estonia. Experience showed, however, that there is a large variance of the tip aperture diameters. It is worth checking several fibres for maximum output divergence.



FIGURE 10: Resolution test of the vertical imaging objective. Images of a SNOM fibre tip (*left*) with distances (x, y) w.r.t. the object plane. The corresponding azimuthally averaged intensity profiles (*right*) are plotted vs the radial coordinate ( $\rho$ ). The red lines are Gaussian fits.

Recorded images of the SNOM fibre tips are shown in Fig. 10, along with the azimuthally averaged spot profiles, fitted by Gaussian functions. The resolution limit  $d_0$  according to the Rayleigh criterion (first zero of the Airy function, Eq. 202) is related to the width  $\sigma$  of the Gaussian that approximates it best by a simple numeric factor,  $d_0 = (1.22/0.42) \sigma$  [303]. For the recorded spots shown, we obtain

$$d_0 = \begin{cases} 0.66(1)\,\mu\text{m} & \text{for} \quad \lambda = 401\,\text{nm}, \\ 0.71(1)\,\mu\text{m} & \text{for} \quad \lambda = 401\,\text{nm}, \end{cases}$$

close to the design values (cf. Table 1 on page 50). The discrepancies are probably mostly due to residual imperfections of the experimental alignment; additionally, there is necessarily a (small) effect of the finite size of the fibre aperture.

Objective mount

Motorised alignment The objective lenses are mounted in a polyetherimide (Ultem<sup>®</sup>) thermoplastic tube, making the assembly inherently non-magnetic and non-conducting. This prevents magnetic hysteresis and induction of eddy currents, which could influence the atomic sample. The objective is mounted from above (see Fig. 11a), with two tilt and three translational degrees of freedom, to allow the necessary fine alignment with respect to the sample. The translational motion in vertical (*z*) direction is driven by a high-resolution stepper motor<sup>31</sup>. In laboratory practice, this is particularly useful for scans to find the exact focus position, or to reproducibly switch between the positions for in-situ and TOF imaging. The clearance from the vacuum window permits to lower the objective by 12 mm at maximum (already excluding a safety spacer) from the in-situ position, allowing times of flight up to around 50 milliseconds without levitation.

The imaging lens can be aligned about five axes with respect to the objective. Our alignment system, home-built from entirely non-magnetic components (Fig. 11b), then allows to move the objective and imaging lens together, without changing their relative alignment. The exact steps of the alignment recipe are described in the appendix, § B.4.

<sup>31</sup> M-229.26S, Physik Instrumente GmbH, Germany.





(a) *xz*-cut through the Er–Dy experiment. Beam paths of in-situ imaging (blue arrows) and vertical MOT beams (yellow/red arrows) are indicated.

(b) The mechanical assembly including the alignment system for the imaging optics.

FIGURE 11: The main-chamber vertical imaging setup.

Example images of quantum gas samples recorded with the newly implemented vertical imaging system are shown in Fig. 12 on the next page, both in in-situ (Faraday phase contrast; a–b) mode and in time-of-flight (absorption; c–d) mode. This new imaging system has by now proved an indispensable tool in the study of quantum droplets and supersolid states of ultracold dipolar atoms, as in our most recent publications (see Refs [207, 272] and §8) as well as in follow-up manuscripts currently in preparation.

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(a) Single-shot Faraday phase contrast in-situ image.



(b) Average image over 20 in-situ shots (without recentring).



(c) Single-shot absorption image after 36 ms TOF.



(d) Average image over 20 TOF shots (without recentring).

FIGURE 12: Images of dipolar 2D supersolids of <sup>164</sup>Dy atoms in trap (top row) and after time-of-flight expansion (TOF, bottom row), imaged with the newly implemented vertical imaging system. Harmonic trap frequencies are  $\omega_{x,y,z} \approx 2\pi \times (51, 49, 134) \text{ s}^{-1}$ , the magnetic field has a magnitude B = 1.92 G and points out of the image plane. The blue-to-red colour map is linear in OD.

Historically, research approaches from two initially independent directions, (i) a quantum-fluctuation-driven stabilisation mechanism and (ii) the dipolar roton dispersion, later joined to form the basis for the field now known as 'quantum droplets and dipolar supersolids'.

Initial interest in the quantum stabilisation mechanism (i) came from theory, triggered by the proposal of Petrov [225] for attractive Bose–Bose mixtures (see § 3.6.1). The quantum droplets predicted by him were realised experimentally not much later in Barcelona [53] and Florence [265], both using mixtures of two hyperfine states of <sup>39</sup>K. The closely related dipolar droplets [17] were observed even earlier by complimentary experiments in Stuttgart [91, 259] and in our group [65]. Whereas the droplets of the Stuttgart group were created by crossing a *sharp phase transition* via a quench of the scattering length at a Feshbach resonance, and therefore highly excited, the macrodroplet studied in our group was obtained by ramping through a *smooth crossover* regime, allowing the preparation of a system much closer to the ground state.

Strong impetus for interest into the roton instability (ii) came, in contrast to the stabilisation mechanism, from experiment, when in 2016 Kadau et al. observed the Rosensweig instability of a <sup>164</sup>Dy BEC in a pancake-shaped trap [145]. This instability manifested in a splitting of the BEC into small, dense, and highly-excited, isolated droplets which the Stuttgart group was able to image in situ. The droplets repelled each other [91] and arranged in a regular array, not dissimilar to the periodic surface structure of a classical ferrofluid beyond the critical magnetisation (see Fig. 5b on page 29). These early droplet arrays were far from the ground state and in addition had a vanishing density overlap between droplets, such that the system as a whole was entirely incoherent [294]. It was at this point completely unclear whether such droplet arrays were a phenomenon caused by and restricted to the excitation of the system, or whether they could also exist in the zero-temperature limit. Nevertheless, it was immediately realised that the breaking of the translational symmetry could open a path to realising a supersolid system from ultracold dipolar atoms.

A key ingredient for the success of later experiments has been the exploration of a cigar-shaped trap geometry in our group [65], where, as mentioned, for an *axial* field the ground state is a macrodroplet. In combination with a *transverse* field, in contrast, the cigar trap has allowed the direct observation of roton mode population in a dipolar BEC [66], since the trap provided a geometric focussing effect in the momentum distribution (Fig. 3 on page 27). Through ground-state calculations based on the extended GPE we now have strong indication that the state in which the early roton population measurements from Ref. [66] had been performed might already have been a supersolid – at the time, however, this was not realised and history took another path.

Soon after the observation of the dipolar Rosensweig instability in Stuttgart it had been speculated whether by a less violent, carefully tuned interaction quench it might be possi-

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ble to realise a gently modulated state with sufficient density overlap between the droplets to maintain coherence. The first experimental step in this direction was made 2019 in the Modugno group in Pisa [279], working with the <sup>162</sup>Dy isotope in a cigar-shaped trap. In their experiment, they observed a density modulation along the cigar axis, which developped in about 10 ms and – for the first time – coexisted with global coherence for about 20 ms, limited by the short lifetime of their samples due to strong three-body loss.

Shortly after, similar results were published by the groups in Stuttgart [44] with <sup>162</sup>Dy and by our group [67] with both, <sup>166</sup>Er and <sup>164</sup>Dy. While the state with transient supersolid properties in the Stuttgart experiment suffered from a limited lifetime of about 20 ms as in Pisa, our group, working with the isotope <sup>164</sup>Dy, was able to realise a system which maintained phase-coherent modulation for more than 150 ms (see § 7) and therefore, for the first time, long enough to actually perform manipulations of the supersolid state. Long parameter scans on the experiment in combination with ground-state calculations using code developped in our group by Rick van Bijnen and Gabriele Natale [66, 205] had been necessary to identify a parameter regime where density modulation is already observable, but peak densities not yet too high, to prevent severe, lifetime-limiting losses. In particular, a careful setting of the trap parameters, atom numbers as high as possible, and a precise tuning of the s-wave scattering length to the level of 1 *a*<sub>0</sub> were required for our measurements presented in Ref [67].

Figure 13 shows examples for the calculated ground states of  $^{164}$ Dy atoms in (a) the BEC as well as (b) the supersolid regime.



FIGURE 13: Zero-temperature ground states for  $7 \times 10^4$  <sup>164</sup>Dy atoms in a harmonic trap of frequencies ( $\omega_x, \omega_y, \omega_z$ ) =  $2\pi \times (128, 72, 30) \text{ s}^{-1}$  at two different s-wave scattering lengths  $a_s$ , calculated from an extended GPE. The plots show the density isosurfaces at both, 1% and 50%, of the respective peak density.

Moreover, in the same publication we were able demonstrate the production of a supersolid state of <sup>164</sup>Dy by direct evaporation from a thermal sample, instead of quenching the interactions in a BEC. This was the first strong experimental indication that not only a highly excited quench product could feature density modulation paired with phase coherence, but also a sample in thermal equilibrium and much closer to the ground state.

The promising results of these measurements motivated us, on the one hand, for a more detailed, dedicated study of the supersolid formation via evaporation [272]. This proved very insightful, since, in contrast to the zero-temperature phase diagram, the supersolid phase and the transition into it from a thermal gas was and still is poorly understood due

to a lack of adequate theory. In our study, we investigate the roles played by condensate atom number and temperature in the evaporative formation of the supersolid, and observe that the translational symmetry is broken before the phase symmetry. These results are presented in §8 and Ref. [272].

On the other hand, the comparatively long lifetime of more than a hundred milliseconds for the supersolid state of  $^{164}$ Dy opened the gates for a follow-up study of the dynamics of supersolid state when it is brought out of equilibrium. In the resulting publication, we were able to demonstrate how the quantum droplets within an array dephase when the density overlap between them is decreased, and how the phases re-lock if the density overlap is replenished. See § 9 and Ref. [143].

Other important experimental studies of the dipolar supersolid have concentrated on its excitation spectrum, mainly driven by the objective to demonstrate its superfluidity directly, and not having to rely on the 'indirect' evidence through the system's global phase coherence, which becomes accessible after a time-of-flight expansion. These studies included the measurement of the Goldstone mode in a supersolid in Stuttgart [126] accompanied by a theoretical study [132], as well as a PCA-based measurement of the roton mode [133], the measurement of compressional oscillations and hints of non-classical rotational inertia in Pisa [280, 281], and works of our group, combining theory and Bragg scattering experiments on the low- [205] and high-energy [228] excitation spectrum of dipolar supersolids of erbium.

Until this point, all experimentally produced dipolar supersolids had been 3D systems in terms of their excitation spectrum, but had shown density modulation only along one direction, the major axis of a cigar-shaped trap. Such a 1D supersolid typically features a line of not more than four to six density peaks, which colloquially are continued to be called 'droplets', even if they have a density connection to their neighbours and are not fully self-bound. The logical but difficult next step was to extend such a system with density modulation along one dimension to a system with density modulation along two independent directions in the plane – 2D supersolidity. Also the Stuttgart group has been pursuing this goal and published several theoretical studies [135, 136, 258]. However, in their experiment they were limited by condensate atom numbers and so supersolidity in 2D stayed beyond reach for them.<sup>1</sup>

In 2021, finally, our group has found a suitable parameter regime and managed to tune up the condensate atom numbers far enough to be able to observe evidence for a departure from the strict 1D supersolidity – a linear-to-zigzag transition of a supersolid of  $^{164}$ Dy atoms. Increased efforts led to the observation of true 2D supersolidity (see Fig. 12 on page 54 ), beyond the zigzag state, and a publication [207] in close collaboration with theorists Russell Bisset (Innsbruck) and Luis Santos (Hannover), who developped a fast analytical simulation model. Further manuscripts on the excitation spectrum of a 2D supersolid, which turns out to be much more complex than in the 1D case, are currently in preparation.

<sup>1</sup> Tilman Pfau, private communication.

# 7

## LONG-LIVED AND TRANSIENT SUPERSOLID BEHAVIORS IN DIPOLAR QUANTUM GASES

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For this publication, I (M.S.) contributed to works on the experiment, analysed the dysprosium data together with A.T. and C.P., and contributed to the interpretation of results well as in writing of the manuscript together with all other authors.

#### PHYSICAL REVIEW X 9, 021012 (2019)

Featured in Physics

#### Long-Lived and Transient Supersolid Behaviors in Dipolar Quantum Gases

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By combining theory and experiments, we demonstrate that dipolar quantum gases of both <sup>166</sup>Er and <sup>164</sup>Dy support a state with supersolid properties, where a spontaneous density modulation and a global phase coherence coexist. This paradoxical state occurs in a well-defined parameter range, separating the phases of a regular Bose-Einstein condensate and of an insulating droplet array, and is rooted in the roton mode softening, on the one side, and in the stabilization driven by quantum fluctuations, on the other side. Here, we identify the parameter regime for each of the three phases. In the experiment, we rely on a detailed analysis of the interference patterns resulting from the free expansion of the gas, quantifying both its density modulation and its global phase coherence. Reaching the phases via a slow interaction tuning, starting from a stable condensate, we observe that <sup>166</sup>Er and <sup>164</sup>Dy exhibit a striking difference in the lifetime of the supersolid properties, due to the different atom loss rates in the two systems. Indeed, while in <sup>166</sup>Er the supersolid behavior survives only a few tens of milliseconds, we observe coherent density modulations for more than 150 ms in <sup>164</sup>Dy. Building on this long lifetime, we demonstrate an alternative path to reach the supersolid regime, relying solely on evaporative cooling starting from a thermal gas.

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#### I. INTRODUCTION

Supersolidity is a paradoxical quantum phase of matter where both crystalline and superfluid order coexist [1–3]. Such a counterintuitive phase, featuring rather antithetic properties, has been originally considered for quantum crystals with mobile bosonic vacancies, the latter being responsible for the superfluid order. Solid <sup>4</sup>He has long been considered a prime system to observe such a phenomenon [4,5]. However, after decades of theoretical and experimental efforts, an unambiguous proof of supersolidity in solid <sup>4</sup>He is still missing [6,7].

In search of more favorable and controllable systems, ultracold atoms emerged as a very promising candidate, thanks to their highly tunable interactions. Theoretical works point to the existence of a supersolid ground state in different cold-atom settings, including dipolar [8]

\*Corresponding author. Francesca.Ferlaino@uibk.ac.at and Rydberg particles [9,10], cold atoms with a softcore potential [11], or lattice-confined systems [7]. Breakthrough experiments with Bose-Einstein condensates (BECs) coupled to light have recently demonstrated a state with supersolid properties [12,13]. While in these systems indeed two continuous symmetries are broken, the crystal periodicity is set by the laser wavelength, making the supersolid incompressible.

Another key notion concerns the close relation between a possible transition to a supersolid ground state and the existence of a local energy minimum at large momentum in the excitation spectrum of a nonmodulated superfluid, known as the roton mode [14]. Since excitations corresponding to a periodic density modulation at the roton wavelength are energetically favored, the existence of this mode indicates the system's tendency to crystallize [15] and it is predicted to favor a transition to a supersolid ground state [4.5,9].

Remarkably, BECs of highly magnetic atoms, in which the particles interact through the long-range and anisotropic dipole-dipole interaction (DDI), appear to gather several key ingredients for realizing a supersolid phase. First, as predicted more than 15 years ago [16,17] and recently demonstrated in experiments [18,19], the partial attraction in momentum space due to the DDI gives rise to a roton

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minimum. The corresponding excitation energy, i.e., the roton gap, can be tuned in the experiments down to vanishing values. Here, the excitation spectrum softens at the roton momentum and the system becomes unstable. Second, there is a nontrivial interplay between the trap geometry and the phase diagram of a dipolar BEC. For instance, our recent observations have pointed out the advantage of axially elongated trap geometries (i.e., cigar shaped) compared to the typically considered cylindrically symmetric ones (i.e., pancake shaped) in enhancing the visibility of the roton excitation in experiments. Last but not least, while the concept of a fully softened mode is typically related to instabilities and disruption of a coherent quantum phase, groundbreaking works in the quantum-gas community have demonstrated that quantum fluctuations can play a crucial role in stabilizing a dipolar BEC [20-26]. Such a stabilization mechanism enables the existence, beyond the mean-field instability, of a variety of stable ground states, from a single macrodroplet [22,24,27] to striped phases [28], and droplet crystals [29]; see also related works [30-33]. For multidroplet ground states, efforts have been devoted to understanding if a phase coherence among ground-state droplets could be established [28,29]. However, previous experiments with <sup>164</sup>Dy have shown the absence of phase coherence across the droplets [28], probably due to the limited atom numbers.

Droplet ground states, quantum stabilization, and dipolar rotons have caused a huge amount of excitement with very recent advancements adding key pieces of information to the supersolid scenario. The quench experiments in an 166Er BEC at the roton instability have revealed out-ofequilibrium modulated states with an early-time phase coherence over a timescale shorter than a quarter of the oscillation period along the weak-trap axis [18]. In the same work, it has been suggested that the roton softening combined with the quantum stabilization mechanism may open a promising route towards a supersolid ground state. A first confirmation came from a recent theoretical work [34], considering an Er BEC in an infinite elongated trap with periodic boundary conditions and tight transverse confinement. The supersolid phase appears to exist within a narrow region in interaction strength, separating a roton excitation with a vanishing energy and an incoherent assembly of insulating droplets. Almost simultaneously, experiments with 162Dy BECs in a shallow elongated trap, performing a slow tuning of the contact interaction, reported on the production of stripe states with phase coherence persisting up to half of the weak trapping period [35]. More recently, such observations have been confirmed in another <sup>162</sup>Dy experiment [36]. Here, theoretical calculations showed the existence of a phase-coherent droplet ground state, linking the experimental findings to the realization of a state with supersolid properties. The results on <sup>162</sup>Dy show, however, transient supersolid properties whose lifetime is limited by fast inelastic losses

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caused by three-body collisions [35,36]. These realizations raise the crucial question of whether a long-lived or stationary supersolid state can be created despite the usually non-negligble atom losses and the crossing of a discontinuous phase transition, which inherently creates excitations in the system.

In this work, we study both experimentally and theoretically the phase diagram of degenerate gases of highly magnetic atoms beyond the roton softening. Our investigations are carried out using two different experimental setups producing BECs of <sup>166</sup>Er [22,37] and of <sup>164</sup>Dy [38] and rely on a fine-tuning of the contact-interaction strength in both systems. In the regime of interest, these two atomic species have different contact-interaction scattering lengths  $a_s$ , whose precise dependence on the magnetic field is known only for Er [18,22,39], and different three-body-loss rate coefficients. Moreover, Er and Dy possess different magnetic moments  $\mu$  and masses *m*, yielding the dipolar lengths,  $a_{dd} = \mu_0 \mu^2 m / 12 \pi \hbar^2$ , of 65.5 $a_0$  and 131 $a_0$ , respectively. Here,  $\mu_0$  is the vacuum permeability,  $\hbar = h/2\pi$  the reduced Planck constant, and  $a_0$  the Bohr radius. For both systems, we find states showing hallmarks of supersolidity, namely, the coexistence of density modulation and global phase coherence. For such states, we quantify the extent of the  $a_s$  parameter range for their existence and study their lifetime. For <sup>166</sup>Er, we find results very similar to the one recently reported for <sup>162</sup>Dy [35,36], both systems being limited by strong three-body losses, which destroy the supersolid properties in about half of a trap period. However, for <sup>164</sup>Dy, we have identified an advantageous magnetic-field region where losses are very low and large BECs can be created. In this condition, we observe that the supersolid properties persist over a remarkably long time, well exceeding the trap period. Based on such a high stability, we finally demonstrate a novel route to reach the supersolid state, based on evaporative cooling from a thermal gas.

#### **II. THEORETICAL DESCRIPTION**

As a first step in our study of the supersolid phase in dipolar BECs, we compute the ground-state phase diagram for both 166Er and 164Dy quantum gases. The gases are confined in a cigar-shaped harmonic trap, as illustrated in Fig. 1(a). Our theory is based on numerical calculations of the extended Gross-Pitaevskii equation [40], which includes our anisotropic trapping potential, the short-range contact and long-range dipolar interactions at a mean-field level, as well as the first-order beyond-mean-field correction in the form of a Lee-Huang-Yang (LHY) term [18,22–24,27]. We note that, while both the exact strength of the LHY term and its dependence on the gas characteristics are under debate [18,19,25,31,41], the importance of such a term, scaling with a higher power in density, is essential for stabilizing states beyond the mean-field instability [18,25,41]; see also Refs. [8,42-44].



FIG. 1. Phase diagram of an <sup>166</sup>Er and a <sup>164</sup>Dy dipolar BEC in a cigar-shaped trap. (a) Illustration of the trap geometry with atomic dipoles oriented along z. (b) Integrated density profile as a function of  $a_s$  for an <sup>166</sup>Er ground state of  $N = 5 \times 10^4$ . In the color bar, the density scale is upper limited to  $4 \times 10^4 \ \mu m^{-1}$  in order to enhance the visibility in the supersolid regime. (c)–(e) Exemplary density profiles for an insulating droplet state (ID) at  $a_s = 49a_0$ , for a state with supersolid properties (SSP) at  $51a_0$ , and for a BEC at  $52a_0$ , respectively. (f),(g) Phase diagrams for <sup>166</sup>Er and <sup>164</sup>Dy for trap frequencies  $\omega_{x,y,z} = 2\pi \times (227, 31.5, 151)$  and  $2\pi \times (225, 37, 135)$  Hz, respectively. The gray color identifies ground states with a single peak in n(y) of large Gaussian width,  $\sigma_y > 2\ell_y$ . The dark blue region in (f) shows the region where n(y) exhibits a single sharp peak,  $\sigma_y \le 2\ell_y$ , and no density modulation. The red-to-blue color map shows S in the case of a density-modulated n(y). In (g) the color map is upper limited to use the same color code as in (f) and to enhance visibility in the low-N regime. The inset in (g) shows the calculated density profile for <sup>164</sup>Dy at  $N = 7 \times 10^4$  and  $a_s = 91a_0$ .

Our theoretical results are summarized in Fig. 1. By varying the condensed-atom number N and  $a_s$ , the phase diagram shows three very distinct phases. To illustrate them, we first describe the evolution of the integrated in situ density profile n(y) with fixed N for varying  $a_s$ , Fig. 1(b). The first phase, appearing at large  $a_s$ , resembles a regular dilute BEC. It corresponds to a nonmodulated density profile of low peak density and large axial size  $\sigma_v$  exceeding several times the corresponding harmonic oscillator length  $(\ell_v = \sqrt{\hbar/m\omega_v})$ ; see Fig. 1(e) and the region denoted BEC in Figs. 1(f) and 1(g). The second phase appears when decreasing  $a_s$  down to a certain critical value,  $a_s^*$ . Here, the system undergoes an abrupt transition to a periodic density-modulated ground state, consisting of an array of overlapping narrow droplets, each of high peak density. Because the droplets are coupled to each other via a density overlap, later quantified in terms of the link strength S, particles can tunnel from one droplet to a neighboring one, establishing a global phase coherence across the cloud; see Fig. 1(d). Such a phase, in which periodic density modulation and phase coherence coexist, is identified as the supersolid (SSP) one [10,34]; see the SSP region in Figs. 1(f) and 1(g). When further decreasing  $a_s$ , we observe a fast reduction of the density overlap, which eventually vanishes; see Fig. 1(c). Here, the droplets become fully separated. Under realistic experimental conditions, it is expected that the phase relation between such droplets cannot be maintained; see later discussion. We identify this third phase as the one of an insulating droplet (ID) array [27,28,45]; see the ID region in Figs. 1(f) and 1(g). For low N, we find a single droplet of high peak density, as in Refs. [24,27]; see dark blue region in Fig. 1(f). Generally speaking, our calculations show that the number of droplets in the array decreases with lowering  $a_s$  or N. The existence of these three phases (BEC, SSP, ID) is consistent with recent calculations considering an infinitely elongated Er BEC [34] and a cigar-shaped <sup>162</sup>Dy BEC [36], illustrating the generality of this behavior in dipolar gases.

To study the supersolid character of the density-modulated phases, we compute the average of the wave function overlap between neighboring droplets S. As an ansatz to extract S, we use a Gaussian function to describe the wave function of each individual droplet. This is found to be an appropriate description from an analysis of the density profiles of Figs. 1(b)-1(d); see also Ref. [46]. For two droplets at a distance d and of identical Gaussian widths  $\sigma_{y}$ along the array direction, S is simply  $S = \exp(-d^2/4\sigma_v^2)$ . Here, we generalize the computation of the wave function overlap to account for the difference in widths and amplitudes among neighboring droplets. This analysis allows us to distinguish between the two types of modulated ground states, SSP and ID in Figs. 1(f) and 1(g). Within the Josephson-junction picture [47-49], the tunneling rate of atoms between neighboring droplets depends on the wave function overlap, and an estimate for the singleparticle tunneling rate can be derived within the Gaussian approximation [46]; see also Ref. [40]. The ID phase corresponds to vanishingly small values of S, yielding

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tunneling times extremely long compared to any other relevant timescale. In contrast, the supersolid phase is identified by a substantial value of S, with a correspondingly short tunneling time.

As shown in Figs. 1(f) and 1(g), a comparative analysis of the phase diagram for <sup>166</sup>Er and <sup>164</sup>Dy reveals similarities between the two species (see also Ref. [36]). A supersolid phase is found for sufficiently high N, in a narrow region of  $a_s$ , upper bounded by the critical value  $a_s^*(N)$ . For intermediate N,  $a_s^*$  increases with increasing N. We note that, for low N, the nonmodulated BEC evolves directly into a single droplet state for decreasing  $a_s$  [50]. In this case, no supersolid phase is found in between; see also Refs. [24,27]. Despite the general similarities, we see that the supersolid phase for <sup>164</sup>Dy appears for lower atom number than for Er and has a larger extension in  $a_s$ . This is mainly due to the different  $a_{dd}$  and strength of the LHY term. We note that, at large N and for decreasing  $a_s$ , Dy exhibits ground states with a density modulation appearing first in the wings, which then progresses inwards until a substantial modulation over the whole cloud is established [51]; see inset of Fig. 1(g). In this regime, we also observe that  $a_s^*$  decreases with increasing N. These types of states have not been previously reported and, although challenging to access in experiments because of the large N, they deserve further theoretical investigations.

#### III. EXPERIMENTAL SEQUENCE FOR <sup>166</sup>Er AND <sup>164</sup>Dy

To experimentally access the above-discussed physics, we produce dipolar BECs of either <sup>166</sup>Er or <sup>164</sup>Dy atoms. These two systems are created in different setups and below we summarize the main experimental steps; see also Ref. [40]. *Erbium.*—We prepare a stable <sup>166</sup>Er BEC following

the scheme of Ref. [18]. At the end of the preparation, the Er BEC contains about  $N = 8 \times 10^4$  atoms at  $a_s = 64.5a_0$ . The sample is confined in a cigar-shaped optical dipole trap with harmonic frequencies  $\omega_{x,y,z}$  =  $2\pi \times (227, 31.5, 151)$  Hz. A homogeneous magnetic field *B* polarizes the sample along *z* and controls the value of  $a_s$ via a magnetic Feshbach resonance (FR) [18,22,40]. Our measurements start by linearly ramping down  $a_s$  within 20 ms and waiting an additional 15 ms so that  $a_s$  reaches its target value [40]. We note that ramping times between 20 and 60 ms have been tested in the experiment and we do not record a significant difference in the system's behavior. After the 15-ms stabilization time, we then hold the sample for a variable time  $t_h$  before switching off the trap. Finally, we let the cloud expand for 30 ms and perform absorption imaging along the z (vertical) direction, from which we extract the density distribution of the cloud in momentum space,  $n(k_x, k_y)$ .

*Dysprosium.*—The experimental procedure to create a <sup>164</sup>Dy BEC follows the one described in Ref. [38]; see also

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Ref. [40]. Similarly to Er, the Dy BEC is also confined in a cigar-shaped optical dipole trap and a homogeneous magnetic field B sets the quantization axis along z and the value of  $a_s$ . For Dy, we will discuss our results in terms of magnetic field B, since the  $a_s$ -to-B conversion is not well known in the magnetic-field range considered [25,40,41,52]. In a first set of measurements, we first produce a stable BEC of about  $N = 3.5 \times 10^4$  condensed atoms at a magnetic field of B = 2.5 G and then probe the phase diagram by tuning  $a_s$ . Here, before ramping the magnetic field to access the interesting  $a_s$  regions, we slowly increase the power of the trapping beams within 200 ms. The final trap frequencies are  $\omega_{x,y,z} = 2\pi \times$ (300, 16, 222) Hz. After preparing a stable BEC, we ramp B to the desired value within 20 ms and hold the sample for  $t_h$  [40]. In a second set of measurements, we study a completely different approach to reach the supersolid state. As discussed later, here we first prepare a thermal sample at a B value where supersolid properties are observed and then further cool the sample until a transition to a coherent droplet-array state is reached. In both cases, at the end of the experimental sequence, we perform absorption imaging after typically 27 ms of time-of-flight (TOF) expansion. The imaging beam propagates horizontally under an angle  $\alpha$  of  $\approx 45^{\circ}$  with respect to the weak axis of the trap (y). From the TOF images, we thus extract  $n(k_Y, k_z)$  with  $k_Y = \cos(\alpha)k_y + \sin(\alpha)k_x$ .

A special property of <sup>164</sup>Dy is that its background scattering length is smaller than  $a_{dd}$ . This allows us to enter the supersolid regime without the need of setting *B* close to a FR, as is done for <sup>166</sup>Er and <sup>162</sup>Dy, which typically causes severe atom losses due to increased threebody-loss coefficients. In contrast, in the case of <sup>164</sup>Dy, the supersolid regime is reached by ramping *B* away from the FR pole used to produce the stable BEC via evaporative cooling, as the  $a_s$  range of Fig. 1(g) lies close to the background  $a_s$  reported in Ref. [52]; see also Ref. [40]. At the background level, three-body-loss coefficients below  $1.3 \times 10^{-41}$  m<sup>6</sup> s<sup>-1</sup> have been reported for <sup>164</sup>Dy [25].

#### IV. DENSITY MODULATION AND PHASE COHERENCE

The coexistence of density modulation and phase coherence is the key feature that characterizes the supersolid phase and allows us to discriminate it from the BEC and ID cases. To experimentally probe this aspect in our dipolar quantum gases, we record their density distribution after a TOF expansion for various values of  $a_s$  across the phase diagram. As for a BEC in a weak optical lattice [53] or for an array of BECs [54–56], the appearance of interference patterns in the TOF images is associated with a density modulation of the *in situ* atomic distribution. Moreover, the shot-to-shot reproducibility of the patterns (in amplitude and position) and the persistence of fringes in averaged



FIG. 2. Coherence in the interference patterns: measurement and toy model. (a)–(c) Examples of single TOF absorption images at  $t_h = 5$  ms for <sup>166</sup>Er at  $a_s = \{54.7(2), 53.8(2), 53.3(2)\}a_0$ , respectively. Corresponding average pictures for 100 images obtained under the same experimental conditions (d)–(f) and their Fourier transform (FT) profiles (g)–(i). The gray lines show the FT norm  $|\mathcal{F}[n](y)|$  of the individual profiles. The averages,  $n_{\mathcal{M}}$  (blue squares) and  $n_{\Phi}$  (red dots), are fitted to three-Gaussian functions (blue solid line and brown dashed line, respectively). The dotted lines show the components of the total fitted function corresponding to the two side peaks in  $n_{\Phi}$ . (j)–(1) Interference patterns from the toy-model realizations with 100 independent draws using  $N_D = 4$ ,  $d = 2.8 \ \mu m$ ,  $\sigma_y = 0.56 \ \mu m$  (see text) and for different  $\phi_i$  distributions: (j)  $\phi_i = 0$ , (k)  $\phi_i$  normally distributed around 0 with 0.2 $\pi$  standard deviation, (l)  $\phi_i$  uniformly distributed between 0 and  $2\pi$ . (m)–(o) Corresponding FT profiles for the toy model, same color code as (g)–(i).

pictures, obtained from many repeated images taken under the same experimental conditions, reveals the presence of phase coherence across the sample [56].

Figure 2 exemplifies snapshots of the TOF distributions for Er, measured at three different  $a_s$  values; see Figs. 2(a)-2(c). Even if very close in scattering length, the recorded  $n(k_x, k_y)$  shows a dramatic change in behavior. For  $a_s = 54.7(2)a_0$ , we observe a nonmodulated distribution with a density profile characteristic of a dilute BEC. When lowering  $a_s$  to 53.8(2) $a_0$ , we observe the appearance of an interference pattern in the density distribution, consisting of a high central peak and two almost symmetric low-density side peaks [57]. Remarkably, the observed pattern is very reproducible with a high shot-to-shot stability, as shown in the repeated single snapshots and in the average image [Figs. 2(b) and 2(e)]. This behavior indicates a coexistence of density modulation and global phase coherence in the in situ state, as expected in the supersolid phase. This observation is consistent with our previous quench experiments [18] and with the recent <sup>162</sup>Dy experiments [35,36]. When further lowering  $a_s$  to  $53.3(2)a_0$ , complicated patterns develop with fringes varying from shot to shot in number, position, and amplitude, signaling the persistence of in situ density modulation. However, the interference pattern is completely washed out in the averaged density profiles [Fig. 2(f)], pointing to the absence of a global phase coherence. We identify this behavior as the one of ID states.

Toy model-To get an intuitive understanding of the interplay between density modulation and phase coherence and to estimate the role of the different sources of fluctuations in our experiment, we here develop a simple toy model, which is inspired by Ref. [56]; see also Ref. [40]. In our model, the initial state is an array of  $N_D$  droplets containing in total N atoms. Each droplet is described by a one-dimensional Gaussian wave function  $\psi_i(y)$  of amplitude  $\alpha_i$ , phase  $\phi_i$ , width  $\sigma_i$ , and center  $y_i$ . To account for fluctuations in the experiments, we allow  $\alpha_i$ ,  $d_i = y_i - y_{i-1}$ , and  $\sigma_i$  to vary by 10% around their expectation values. The spread of the phases  $\phi_i$  among the droplets is treated specially as it controls the global phase coherence of the array. By fixing  $\phi_i = 0$  for each droplet or by setting a random distribution of  $\phi_i$ , we range from full phase coherence to the incoherent cases. Therefore, the degree of phase incoherence can be varied by changing the standard deviation of the distribution of  $\phi_i$ .

To mimic our experiment, we compute the free evolution of each individual  $\psi_i$  over 30 ms, and then compute the axial distribution  $n(y, t) = |\sum_i \psi_i(y, t)|^2$ , from which we extract the momentum distribution  $n(k_y)$ , also accounting for the finite imaging resolution [40]. For each computation run, we randomly draw  $N_D$  values for  $\phi_i$ , as well as of  $\sigma_i$ ,  $d_i$ , and  $\alpha_i$ , and extract  $n(k_y)$ . We then collect a set of  $n(k_y)$ 

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by drawing these values multiple times using the same statistical parameters and compute the expectation value,  $\langle n(k_y) \rangle$ ; see Figs. 2(j)–2(l). The angled brackets denote the ensemble average.

The results of our toy model show large similarity with the observed behavior in the experiment. In particular, while for each single realization one can clearly distinguish multipeak structures regardless of the degree of phase coherence between the droplets, the visibility of the interference pattern in the averaged  $n(k_y)$  survives only if the standard deviation of the phase fluctuations between droplets is small (roughly, below  $0.3\pi$ ). In the incoherent case, we note that the shape of the patterns strongly varies from shot to shot. Interestingly, the toy model also shows that the visibility of the coherent peaks in the average images is robust against the typical shot-to-shot fluctuations in droplet size, amplitude, and distance that occur in the experiments; see Figs. 2(j) and 2(k).

Probing density modulation and phase coherence.—To separate and quantify the information on the *in situ* density modulation and its phase coherence, we analyze the measured interference patterns in Fourier space [36,58–60]. Here, we extract two distinct averaged density profiles,  $n_M$  and  $n_{\Phi}$ . Their structures at finite *y* spatial frequency (i.e., in Fourier space) quantify the two abovementioned properties.

More precisely, we perform a Fourier transform (FT) of the integrated momentum distributions  $n(k_y)$  denoted  $\mathcal{F}[n](y)$ . Generally speaking, modulations in  $n(k_y)$  induce peaks at finite spatial frequency,  $y = y^*$ , in the FT norm,  $|\mathcal{F}[n](y)|$ ; see Figs. 2(g)–2(i) and 2(m)–2(o). Following the above discussion (see also Refs. [56,61]), such peaks in an individual realization hence reveal a density modulation of the corresponding in situ state, with a wavelength roughly equal to  $y^*$ . Consequently, we consider the average of the FT norm of the individual images,  $n_{\mathcal{M}}(y) = \langle |\mathcal{F}[n](y)| \rangle$ , as the first profile of interest. The peaks of  $n_M$  at finite y then indicate the mere existence of an in situ density modulation of roughly constant spacing within the different realizations. As the second profile of interest, we use the FT norm of the average profile  $\langle n(k_y) \rangle$ ,  $n_{\Phi}(y) = |\mathcal{F}[\langle n \rangle](y)|$ . Connecting to our previous discussion, the peaks of  $n_{\Phi}$  at finite y point to the persistence of a modulation in the average  $\langle n(k_y) \rangle$ , which we identified as a hallmark for a global phase coherence within the density-modulated state. In particular, we point out that a perfect phase coherence, implying identical interference patterns in all the individual realizations, yields  $n_{\mathcal{M}} = n_{\Phi}$  and, thus, identical peaks at finite y in both profiles. We note that, by linearity,  $n_{\Phi}$ also matches the norm of the average of the full FT of the individual images, i.e.,  $n_{\Phi}(y) = |\langle \mathcal{F}[n](y) \rangle|$ ; see also Ref. [40].

Figures 2(g)-2(i) and 2(m)-2(o) demonstrate the significance of our FT analysis scheme by applying it to the momentum distributions from the experiment [Figs. 2(d)-2(f)] and the momentum distributions from

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the toy model [Figs. 2(j)-2(l)], respectively. As expected, for the BEC case, both  $n_{\mathcal{M}}$  and  $n_{\Phi}$  show a single peak at zero spatial frequency, y = 0, characterizing the absence of density modulation, Fig. 2(g). In the case of phase-coherent droplets, Fig. 2(e), we observe that  $n_M$  and  $n_{\Phi}$  are superimposed and both show two symmetric side peaks at finite y, in addition to a dominant peak at y = 0; see Fig. 2(h). In the incoherent droplet case, we find that, while  $n_{\mathcal{M}}$  still shows side peaks at finite y, the ones in  $n_{\Phi}$  wash out from the averaging, Figs. 2(f), 2(i), 2(l), and 2(o). For both coherent and incoherent droplet arrays, the toy-model results show behaviors matching the above description, providing a further justification of our FT analysis scheme; see Figs. 2(j)-2(o). Our toy model additionally proves two interesting features. First, it shows that the equality  $n_{\mathcal{M}} = n_{\Phi}$ , revealing the global phase coherence of a density-modulated state, is remarkably robust to noise in the structure of the droplet arrays; see Figs. 2(j) and 2(m). Second, our toy model, however, shows that phase fluctuations across the droplet array on the order of  $0.2\pi$ standard deviation are already sufficient to make  $n_{\Phi}$  and  $n_{\mathcal{M}}$  deviate from each other; see Figs. 2(k) and 2(n). The incoherent behavior is also associated with strong variations in the side peak amplitude of the individual realizations of  $|\mathcal{F}[n]|$ , connecting, e.g., to the observations of Ref. [36].

Finally, to quantify the density modulation and the phase coherence, we fit a three-Gaussian function to both  $n_{\mathcal{M}}(y)$  and  $n_{\Phi}(y)$  and extract the amplitudes of the finite-spatial-frequency peaks,  $A_{\mathcal{M}}$  and  $A_{\Phi}$ , for both distributions, respectively. Note that for a BEC, which is a phase-coherent state,  $A_{\Phi}$  will be zero since it probes only finite-spatial-frequency peaks; see Figs. 2(g)–2(i) and 2(m)–2(o).

#### V. CHARACTERIZATION OF THE SUPERSOLID STATE

We are now in the position to study two key aspects, namely, (i) the evolution of the density modulation and phase coherence across the BEC-supersolid-ID phases and (ii) the lifetime of the coherent density-modulated state in the supersolid regime.

Evolution of the supersolid properties across the phase diagram.—The first type of investigation is conducted with <sup>166</sup>Er since, contrary to <sup>164</sup>Dy, its scattering length and dependence on the magnetic field has been precisely characterized [18,22]. After preparing the sample, we ramp  $a_s$  to the desired value and study the density patterns as well as their phase coherence by probing the amplitudes  $A_M$ and  $A_{\Phi}$  as a function of  $a_s$  after  $t_h = 5$  ms. As shown in Fig. 3(a), in the BEC region (i.e., for large  $a_s$ ), we observe that both  $A_M$  and  $A_{\Phi}$  are almost zero, evidencing the expected absence of a density modulation in the system. As soon as  $a_s$  reaches a critical value  $a_s^*$ , the system's behavior dramatically changes with a sharp and simultaneous



FIG. 3. Supersolid behavior across the phase diagram. Measured side peak amplitudes,  $A_{\Phi}$  (circles) and  $A_{\mathcal{M}}$  (squares), with their ratio in inset (a), and calculated link strength *S* (b) as a function of  $a_s - a_s^*$  for <sup>166</sup>Er. For nonmodulated states, we set S = 0 in theory and  $A_{\Phi}/A_{\mathcal{M}} = 0$  in experiment (crosses in inset). In the inset, open and closed symbols correspond to  $A_{\Phi}/A_{\mathcal{M}} > 0.8$  and  $\leq 0.8$ , respectively. In the experiments, we probe the system at a fixed  $t_h = 5$  ms. Horizontal error bars are derived from our experimental uncertainty in *B*, vertical error bars corresponding to the statistical uncertainty from the fit are smaller than the data points. The measured and calculated critical scattering lengths are  $a_s^* = 54.9(2)a_0$  and  $51.15a_0$ , respectively [62]. The numerical results are obtained for the experimental trap frequencies and for a constant  $N = 5 \times 10^4$  [63].

increase of both  $A_{\mathcal{M}}$  and  $A_{\Phi}$ . While the strength of  $A_{\mathcal{M}}$ and  $A_{\Phi}$  varies with decreasing  $a_s$ —first increasing then decreasing—we observe that their ratio  $A_{\Phi}/A_{\mathcal{M}}$  remains constant and close to unity over a narrow  $a_s$  range below  $a_s^*$ of  $\gtrsim 1a_0$  width; see the inset of Fig. 3(a). This behavior pinpoints the coexistence in the system of phase coherence and density modulation, as predicted to occur in the supersolid regime. For  $(a_s - a_s^*) < -1a_0$ , we observe that the two amplitudes depart from each other. Here, while the density modulation still survives with  $A_{\mathcal{M}}$  saturating to a lower finite value, the global phase coherence is lost with  $A_{\Phi}/A_{\mathcal{M}} < 1$ , as expected in the insulating droplet phase. Note that we also study the evolution of  $A_{\Phi}$  and  $A_{\mathcal{M}}$  in <sup>164</sup>Dy, but as a function of B, and find a qualitatively similar behavior.

To get a deeper insight on how our observations compare to the phase-diagram predictions (see Fig. 1), we study the link strength *S* as a function of  $a_s$ ; see Fig. 3(b). Since *S* quantifies the density overlap between neighboring droplets and is related to the tunneling rate of atoms across the droplet array, it thus provides information on the ability of

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the system to establish or maintain a global phase coherence. In this plot, we set S = 0 in the case where no modulation is found in the ground state. At the BEC-tosupersolid transition, i.e., at  $a_s = a_s^*$ , a density modulation abruptly appears in the system's ground state with S taking a finite value. Here, S is maximal, corresponding to a density modulation of minimal amplitude. Below the transition, we observe a progressive decrease of S with lowering  $a_s$ , pointing to the gradual reduction of the tunneling rate in the droplet arrays. Close to the transition, we estimate a large tunneling compared to all other relevant timescales. However, we expect this rate to become vanishingly small, on the sub-Hertz level [40], when decreasing  $a_s 1-2a_0$  below  $a_s^*$ . Our observation also hints at the smooth character of the transition from a supersolid to an ID phase.

The general trend of S, including the extension in  $a_s$ where it takes nonvanishing values, is similar to the  $a_s$ behavior of  $A_{\mathcal{M}}$  and  $A_{\Phi}$  observed in the experiments [62]. We observe in the experiments that the  $a_s$  dependence at the BEC-to-supersolid transition appears sharper than at the supersolid-to-ID interface, potentially suggesting a different nature of the two transitions. However, more investigations are needed since atom losses, finite temperature, and finite-size effects can affect, and in particular smoothen, the observed behavior [64-66]. Moreover, dynamical effects, induced by, e.g., excitations created at the crossing of the phase transitions or atom losses during the time evolution, can also play a substantial role in the experimental observations, complicating a direct comparison with the ground-state calculations. The time dynamics as well as a different scheme to achieve a state with supersolid properties is the focus of the remainder of the paper.

Lifetime of the supersolid properties.—Having identified the  $a_s$  range in which our dipolar quantum gas exhibits supersolid properties, the next central question concerns the stability and lifetime of such a fascinating state. Recent experiments on <sup>162</sup>Dy have shown the transient character of the supersolid properties, whose lifetime is limited by three-body losses [35,36]. In these experiments, the phase coherence is found to survive up to 20 ms after the density modulation has formed. This time corresponds to about half of the weak-trap period. Stability is a key issue in the supersolid regime, has a twofold consequence on the inelastic loss rate. First, it gives rise to an increase in the peak density [see Figs. 1(b)–1(d)] and, second, it may lead to an enhancement of the three-body-loss coefficient.

We address this question by conducting comparative studies on <sup>166</sup>Er and <sup>164</sup>Dy gases. These two species allow us to tackle two substantially different scattering scenarios. Indeed, the background value of  $a_s$  for <sup>166</sup>Er (as well as for <sup>162</sup>Dy) is larger than  $a_{dd}$ . Thus, reaching the supersolid regime, which occurs at  $a_{dd}/a_s \approx 1.2$ –1.4 in our geometry, requires us to tune *B* close to the pole of a FR. This tuning

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also causes an increase of the three-body-loss rate. In contrast, <sup>164</sup>Dy realizes the opposite case with the background scattering length smaller than  $a_{dd}$ . This feature brings the important advantage of requiring tuning *B* away from the FR pole to reach the supersolid regime. As we describe below, this important difference in scattering properties leads to a strikingly longer lifetime of the <sup>164</sup>Dy supersolid properties with respect to <sup>166</sup>Er and to the recently observed behavior in <sup>162</sup>Dy [35,36].

The measurements proceed as follows. For both <sup>166</sup>Er and <sup>164</sup>Dy, we first prepare the quantum gas in the stable BEC regime and then ramp  $a_s$  to a fixed value in the supersolid regime for which the system exhibits a state of coherent droplets (i.e.,  $A_{\Phi}/A_{\mathcal{M}} \approx 1$ ); see previous discussion. Finally, we record the TOF images after a variable  $t_h$  and we extract the time evolution of both  $A_{\Phi}$  and  $A_{\mathcal{M}}$ . The study of these two amplitudes will allow us to answer the question of whether the droplet structure—i.e., the density modulation in space—persists in time whereas the coherence among droplets is lost  $(A_{\mathcal{M}} > A_{\Phi} \to 0)$  or if the density structures themselves vanish in time  $(A_{\mathcal{M}} \approx A_{\Phi} \to 0)$ .

As shown in Fig. 4, for both species, we observe that  $A_{\Phi}$  and  $A_{\mathcal{M}}$  decay almost synchronously with a remarkably longer lifetime for <sup>164</sup>Dy [Fig. 4(b)] than <sup>166</sup>Er [Fig. 4(a)].



FIG. 4. Time evolution of the supersolid properties. Amplitudes  $A_{\Phi}$  (circles) and  $A_{\mathcal{M}}$  (squares) in the supersolid regime as a function of the holding time in trap for (a) <sup>166</sup>Er at 54.2(2) $a_0$  and for (b) <sup>164</sup>Dy at 2.04 G. The solid lines are exponential fits to the data. The insets show the time evolution of  $A_{\Phi}/A_{\mathcal{M}}$  for the above cases (filled triangles), and, for comparison, in the ID regime (empty triangles) for Er at  $a_s = 53.1(2)a_0$  (a).

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Interestingly,  $A_{\Phi}$  and  $A_{\mathcal{M}}$  remain approximately equal during the whole time dynamics; see insets of Figs. 4(a) and 4(b). This behavior indicates that it is the strength of the density modulation itself and not the phase coherence among droplets that decays over time. Similar results have been found theoretically in Ref. [67]. We connect this decay mainly to three-body losses, especially detrimental for <sup>166</sup>Er, and possible excitations created while crossing the BEC-to-supersolid phase transition [40]. For comparison, the inset of Fig. 4(a) shows also the behavior in the ID regime for <sup>166</sup>Er, where  $A_{\Phi}/A_{\mathcal{M}} < 1$  already at short  $t_h$  and remains so during the time evolution [40].

To get a quantitative estimate of the survival time of the phase-coherent and density-modulated state, we fit a simple exponential function to  $A_{\Phi}$  and extract  $t_{\Phi}$ , defined as the 1/10 lifetime; see Fig. 4. For <sup>166</sup>Er, we extract  $t_{\Phi} = 38(6)$  ms. For  $t_h > t_{\Phi}$ , the interference patterns become undetectable in our experiment and we recover a signal similar to the one of a nonmodulated BEC state [as in Figs. 2(a) and 2(d)]. These results are consistent with recent observations of transient supersolid properties in <sup>162</sup>Dy [35]. For <sup>164</sup>Dy, we observe that the coherent density-modulated state is remarkably long-lived. Here, we find  $t_{\Phi} = 152(13)$  ms.

The striking difference in the lifetime and robustness of the supersolid properties between <sup>166</sup>Er and <sup>164</sup>Dy becomes even more visible when studying  $t_{\Phi}$  as a function of  $a_s$ (B for Dy). As shown in Fig. 5,  $t_{\Phi}$  for Er remains comparatively low in the investigated supersolid regime and slightly varies between 20 and 40 ms. Similarly to the recent studies with 162Dy, this finding reveals the transient character of the state and opens the question of whether a stationary supersolid state can be reached with these species. On the contrary, for  $^{164}$ Dy we observe that  $t_{\Phi}$ first increases with B in the range from 1.8 G to about 1.98 G. Then, for B > 1.98 G,  $t_{\Phi}$  acquires a remarkably large and almost constant value of about 150 ms over a wide B range. This shows the long-lived character of the supersolid properties in our <sup>164</sup>Dy quantum gas. We note that over the investigated range,  $a_s$  is expected to monotonously increase with B [40]. Such a large value of  $t_{\Phi}$ exceeds not only the estimated tunneling time across neighboring droplets but also the weak-axis trap period, which together set the typical timescale to achieve global equilibrium and to study collective excitations.

#### VI. CREATION OF STATES WITH SUPERSOLID PROPERTIES BY EVAPORATIVE COOLING

The long-lived supersolid properties in <sup>164</sup>Dy motivate us to explore an alternative route to cross the supersolid phase transition, namely, by evaporative cooling instead of interaction tuning. For this set of experiments, we have modified the waists of our trapping beams in order to achieve quantum degeneracy in tighter traps with respect to



FIG. 5. Survival time of the coherent density-modulated state.  $t_{\Phi}$  in <sup>166</sup>Er as a function of  $a_s$  (a) and <sup>164</sup>Dy as a function of *B* (b). The error bars refer to the statistical uncertainty from the fit. The range of investigation corresponds to the supersolid regime for which phase-coherent density-modulated states are observed. This range is particularly narrow for <sup>166</sup>Er.

the one used for condensation in the previous set of measurements. In this way, the interference peaks in the supersolid region are already visible without the need to apply a further compression of the trap since the side-to-central-peak distance in the momentum distribution scales roughly as  $1/\ell_z$  [18]. Forced evaporative cooling is performed by reducing the power of the trapping beams piecewise linearly in subsequent evaporation steps until a final trap with frequencies  $2\pi \times (225, 37, 134)$  Hz is achieved. During the whole evaporation process, which has an overall duration of about 3 s, the magnetic field is kept either at B = 2.43 G, where we observe long-lived interference patterns, or at B = 2.55 G, where we produce a stable nonmodulated BEC. We note that these two *B* values are very close without any FR lying in between [40].

Figure 6 shows the phase transition from a thermal cloud to a final state with supersolid properties by evaporative cooling. In particular, we study the phase transition by varying the duration of the last evaporation ramp, while maintaining the initial and final trap-beam power fixed. This procedure effectively changes the atom number and temperature in the final trap while keeping the trap parameters unchanged, which is important to not alter the final ground-state phase diagram of the system. At the end of the evaporation, we let the system equilibrate and thermalize for  $t_h = 100$  ms, after which we switch off the trap, let the atoms expand for 26.5 ms, and finally perform absorption imaging. We record the TOF images for different ramp durations, i.e., for different thermalization times. For a short ramp, too many atoms are lost such that the critical atom number for condensation is not reached, and the atomic distribution remains thermal; see Fig. 6(a).



FIG. 6. Evaporative cooling to a state with supersolid properties. <sup>164</sup>Dy absorption images showing the transition to a state with supersolid properties at 2.43 G (a)–(d) and to a BEC state at 2.55 G (i)–(l), via different durations of the last evaporation step. These durations are 10 ms (a),(i), 50 ms (b),(j), 100 ms (c),(k), and 300 ms (d),(l). The density profiles (e)–(h) are integrated over the central regions of the corresponding absorption images (a)–(d). The color map indicates the atomic density in momentum space.

By increasing the ramp time, the evaporative cooling becomes more efficient and we observe the appearance of a bimodal density profile with a narrow and dense peak at the center, which we identify as a regular BEC; see Fig. 6(b). By further cooling, the BEC fraction increases and the characteristic pattern of the supersolid state emerges; see Figs. 6(c) and 6(d). The observed evaporation process shows a strikingly different behavior in comparison with the corresponding situation at B = 2.55 G, where the usual thermal-to-BEC phase transition is observed; see Figs. 6(i)–6(1).

We finally probe the lifetime of the supersolid properties by extracting the time evolution of both the amplitudes  $A_{\Phi}$ and  $A_{\mathcal{M}}$ , as previously discussed. We use the same experimental sequence as the one in Fig. 6(d)—i.e., 300ms duration of the last evaporation ramp and 100 ms of equilibration time—and subsequently hold the sample in the trap for a variable  $t_h$ . As shown in Fig. 7(a), we observe a very long lifetime with both amplitudes staying large and almost constant over more than 200 ms. At longer holding

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FIG. 7. Lifetime of the supersolid properties achieved via evaporative cooling. Time evolution of the amplitudes  $A_{\Phi}$  (red circle) and  $A_{\mathcal{M}}$  (square) after an evaporation time of 300 ms at 2.43 G and an equilibration time of 100 ms. The inset shows the time evolution of  $A_{\Phi}/A_{\mathcal{M}}$ . At  $t_h = 0$  ms, the atom number in the phase-coherent density-modulated component is  $N = 2.2(2) \times 10^4$ . (b),(c) Averaged absorption images of 25 realizations after 50 and 300 ms of holding time, respectively. Note that the thermal background has been subtracted from the images. The color map indicates the atomic density in momentum space.

time, we observe a slow decay of  $A_{\Phi}$  and  $A_{\mathcal{M}}$ , following the decay of the atom number. Moreover, during the dynamics, the ratio  $A_{\Phi}/A_{\mathcal{M}}$  stays constant. The long lifetime of the phase-coherent density modulation is also directly visible in the persistence of the interference patterns in the averaged momentum density profiles [similar to Fig. 2(e)], both at intermediate and long times; see Figs. 7(b) and 7(c), respectively. For even longer  $t_h$ , we cannot resolve anymore interference patterns in the TOF images. Here, we recover a signal consistent with a regular BEC of low N.

Achieving the coherent droplet phase via evaporative cooling is a very powerful alternative path to supersolidity. We speculate that, for instance, excitations, which might be important when crossing the phase transitions by interaction tuning, may be small or removed by evaporation when reaching this state kinematically. Other interesting questions, open to future investigations, are the nature of the phase transition, the critical atom number, and the role of noncondensed atoms.

#### **VII. CONCLUSIONS**

For both <sup>166</sup>Er and <sup>164</sup>Dy dipolar quantum gases, we have identified and studied states showing hallmarks of supersolidity, namely, global phase coherence and spontaneous density modulations. These states exist in a narrow scattering-length region, lying between a regular BEC phase and a phase of an insulating droplet array. While for <sup>166</sup>Er, similarly to the recently reported <sup>162</sup>Dy case [35,36], the observed supersolid properties fade out over a comparatively short time because of atom losses, we find that <sup>164</sup>Dy exhibits remarkably long-lived supersolid properties. Moreover, we are able to directly create stationary states

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with supersolid properties by evaporative cooling, demonstrating a powerful alternative approach to interaction tuning on a BEC. This novel technique provides prospects of creating states with supersolid properties while avoiding additional excitations and dynamics. The ability to produce long-lived supersolid states paves the way for future investigations on quantum fluctuations and many-body correlations, as well as of collective excitations in such an intriguing many-body quantum state. A central goal of these future investigations lies in proving the superfluid character of this phase, beyond its global phase coherence [7,34,68–70].

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*Note added.*—Recently, we became aware of related works reporting theoretical studies of the ground-state phase diagram [71,72].

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*Correction:* The inadvertent omission of a marker indicating "Featured in Physics" has been fixed.

## Supplemental Material: Long-lived and transient supersolid behaviors in dipolar quantum gases

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GROUND STATE CALCULATIONS

We perform numerical calculations of the ground state following the procedure detailed in the supplementary information of Ref. [1]. The calculations are based on the conjugate-gradients technique to minimize the energy functional of an eGPE [2]. In particular, the eGPE accounts for the effect of quantum fluctuations, by including the LHY term  $\Delta \mu[n] = 32 g (n a_{\rm s})^{3/2} (1 + 3 \epsilon_{\rm dd}^2/2)/3 \sqrt{\pi}$ in the system's Hamiltonian (here  $g = 4\pi \hbar^2 a_s/m$  and  $n = |\psi|^2$  is the spatial density of the macroscopic state  $\psi$ ).  $\Delta \mu[n]$  has been obtained under a local density approximation in Refs. [3, 4]. The relevance of the LHY correction has been demonstrated in various studies of dipolar Bose gases close to the mean-field instability [1, 5-9] as it brings an additional repulsive potential. stabilizing the gas against mean-field collapse at large density. We note that the exact functional form of the potential, originating from beyond mean-field effects, has been questioned by several experimental results in finitesize trapped systems [1, 9–11], calling for further theory developments [12].

Our numerical calculations provide us with the threedimensional ground-state wavefunctions  $\psi(\mathbf{r})$ . From this, we compute the axial in-situ density profile along the trap's weak axis,  $n(y) = \int |\psi(\mathbf{r})|^2 dx dz$  and find density profiles, corresponding to the BEC, the supersolid or the ID phase, that we plot in Fig.1. From the density profiles that exhibit a density modulation, we evaluate S by performing Gaussian fits to each droplet, i.e. to n(y) with y ranging between two neighboring local density minima. From these Gaussian fits, we evaluate the sets of centers  $\{y_i^{(0)}\}_i$  and widths  $\{\sigma_i\}_i$  corresponding to the macroscopic Gaussian wavefunctions  $\{\psi_i\}_i$  associated to the individual droplets in the array. We then approximate the droplet wavefunction via  $\psi_i(y) \approx \sqrt{n(y \approx y_i^{(0)})} = \alpha_i \exp\left(-(y - y_i^{(0)})^2/2\sigma_i^2\right)$  with  $\alpha_i$  a normalization coefficient such that  $\int |\psi_i(y)|^2 dy = 1$ . We then evaluate the wavefunction overlap  $S_i$  between

the neighboring droplets i - 1 and i via:

$$S_{i} \equiv \int \psi_{i-1}^{*}(y)\psi_{i}(y)dy$$
(1)  
=  $\sqrt{\frac{2\sigma_{i}\sigma_{i-1}}{\sigma_{i}^{2} + \sigma_{i-1}^{2}}} \exp\left(-\frac{(y_{i}^{(0)} - y_{i-1}^{(0)})^{2}}{2(\sigma_{i}^{2} + \sigma_{i-1}^{2})}\right).$ (2)

The latter equation is obtained via an analytical evaluation of the Gaussian integral. The characteristic link strength defined in the paper is then computed by averaging  $S_i$  over all droplet links in the array:  $S = \langle S_i \rangle_i$ . In our calculation, we only consider as droplets all density peaks of at least 5% of the global density maximum.

#### LINK STRENGTH AND ESTIMATE OF TUNNELING RATE

Generally speaking, the wavefunction overlap between neighboring droplets relates to a tunneling term, which sets a particle exchange term between two neighboring droplets  $[13\mathchar`-16].$  Following the work of Ref. [17], we perform a first estimate of the tunneling coefficient by simply considering the single-particle part of the Hamiltonian and evaluate it between two neighboring droplets. We note that, in our particular setting where the density modulation is not externally imposed but arises from the mere interparticle interactions, the inter-droplet interaction may also play a crucial role. To perform a more precise estimation of the tunneling between droplets, one would certainly need to properly account for this effect. Here, we stress that our approach simply gives a rough idea of the magnitude of tunneling while it does not aim to be a quantitative description of it. This consideration calls for further studies making a systematic analysis of the full Hamiltonian and of the full phase diagram within the Josephson junction formalism and beyond.

Generalizing the description of Ref. [17] to neighboring droplets of different sizes and amplitudes, which are described by a three-dimensional wavefunction  $\psi_i(\mathbf{r})$  approximated to a three-dimensional Gaussian of widths  $(\sigma_{i,x}, \sigma_{i,y}, \sigma_{i,z})$  with  $\sigma_{i,y} = \sigma_i$ , our estimate writes:

$$\begin{split} J_{i} &= \frac{\hbar^{2}S_{i}}{2m} \left[ \sum_{k=x,y,z} \frac{1 + \left(\frac{\sigma_{i,k}\sigma_{i-1,k}}{\ell_{k}^{2}}\right)^{2}}{\sigma_{i,k}^{2} + \sigma_{i-1,k}^{2}} \right. \\ &+ \frac{(y_{i}^{(0)} - y_{i-1}^{(0)})^{2}}{2\sigma_{i}\sigma_{i-1}} \frac{(\sigma_{i}\sigma_{i-1}/\ell_{y})^{4} - 1}{\sigma_{i}^{2} + \sigma_{i-1}^{2}} \right], \qquad (3)$$

where  $\ell_{x,y,z} = \sqrt{\hbar/m\omega_{x,y,z}}$  are the harmonic oscillator lengths.

In general, the tunnelling coefficients set two typical rates relevant for equilibration processes. The first one is the bare single-particle tunneling rate, which is equal to  $J_i/h$ , while the second accounts for the bosonic enhancement from the occupation of the droplet modes and writes  $\tilde{t}_i = \sqrt{N_i N_{i-1}} |J_i|/h$  where  $N_i$  is the number of atoms in droplet *i*. In our analysis, we then define the average rates over the droplet arrays as characteristic rates  $J/h = \langle J_i \rangle_i/h$ , and  $\tilde{t} = \langle \tilde{t}_i \rangle_i$ ; see e.g. [18]. While the ground state evolves from a BEC to a supersolid to an ID, the relevant timescale for achieving (global) equilibrium crosses from being set by the trap frequencies to the above-mentioned tunneling rates.

Using our approximate model, we here give a first estimate of the rates J/h and  $\tilde{t}$  as a function of  $a_{\rm s}$ , for the parameters of Fig. 1(b-d) of the main text (i.e. Er quantum gas with  $N = 5 \times 10^4$  atoms). Here we find that, for  $a_{\rm s} = a_{\rm s}^*$ ,  $J/h \sim 400 \,{\rm Hz}$  and  $\tilde{t} \sim 10 \,{\rm MHz}$  while for  $a_{\rm s} = a_{\rm s}^* - 2.5 \,a_0$ ,  $J/h \sim 10^{-7} \,{\rm Hz}$  and  $\tilde{t} \sim 10^{-3} \,{\rm Hz}$ .

#### TOY MODEL FOR THE INTERFERENCE PATTERN

As described in the main text we use a simple toy model, adapted from Ref. [18], to identify the main features of the TOF interference patterns obtained from an insitu density-modulated state. As a quick reminder, our model considers a one-dimensional array of  $N_D$  Gaussian droplets, described by a single classical field,  $\psi_i$ , thus neglecting quantum and thermal fluctuations. We compute the TOF density distribution from the freeexpansion of the individual  $\psi_i$  during a time t via  $n(y,t) = |\sum_i \psi_i(y,t)|^2$ . In our calculations, we also account for the finite imaging resolution by convolving the resulting n(y,t) with a gaussian function of width  $\sigma_{\rm im}$ . Here we allow the characteristics of the individual  $\psi_i$  to fluctuate. In this aim, we introduce noise on the corresponding parameter with a normal distribution around its expectation value and with a variable standard deviation (only  $\phi_i$  can also have a uniform distribution). We then perform a Monte-Carlo study and perform ensemble averages, similar to our experimental analysis procedure. We note that, in this simple implementation, the noise on the different parameters - droplet amplitudes, widths and distances - are uncorrelated.

In the main text, we present results for a single set of parameters, namely  $N_D = 4$ ,  $d \equiv \langle d_i \rangle_i = 2.8 \,\mu\text{m}$  (mean droplet distance),  $\sigma_y \equiv \langle \sigma_i \rangle_i = 0.56 \,\mu\text{m}$  (mean droplet size),  $t = 30 \,\text{ms}$ , and  $\sigma_{\text{im}} = 3 \,\mu\text{m}$ , typical for our experimental Er setting and the corresponding theory expectations in the supersolid regime.  $\langle \cdot \rangle_i$  denotes the average over the droplets. In this section, we have a deeper look at the impact of the different parameters on both the TOF signal and our FT analysis. We study both the fully phase coherent and fully incoherent case, and the unchanged parameters are set as in Fig. 2(j,m) and (l,o).



FIG. S1. Toy model realizations with varying number of droplets  $N_D$ . We use 100 independent draws, and expectation values  $d = 2.85 \,\mu\text{m}$ ,  $\sigma_y = 0.56 \,\mu\text{m}$  (with 10% noise) and either  $\phi_i = 0$  (a,b,e,f,i,j), or  $\phi_i$  uniformly distributed between 0 and  $2\pi$  (c,d,g,h,k,l). (a–d)  $N_D = 2$ , (e–h)  $N_D = 3$  and (i–l)  $N_D = 8$ . (a,c,e,g,i,k) TOF density profiles and (b,d,f,h,j,l) corresponding FT analysis of the interference patterns, same color code as Fig.2.

In Fig. S1, we first exemplify the TOF and FT profiles for a varying number of droplets, between 2 and 8, which cover the range of relevant  $N_D$  over the phase diagram of Fig.1. The results remain remarkably similar to the realization of Fig.2 with only slight quantitative changes. The main difference lies in the individual interference patterns obtained in the phase incoherent case. With increasing  $N_D$ , those profiles become more complex and made of a larger number of peaks (see (c,g,k)). Yet, in this incoherent case, a similar (non-modulated) profile is recovered in the averaged  $n(k_y)$  for all  $N_D$ . Additionally, we note that for the coherent case with  $N_D = 8$ , the side peaks in the FT analysis (see (j)) become less visible. By performing additional tests, we attribute this behavior to the limited TOF duration, t, used in our experiment vielding a typical length scale.  $\sqrt{\hbar t/m}$  (= 3.4µm), which becomes small compared to the system size  $(\approx (N_D - 1)d + \sigma_y)$  for large  $N_D$ . This intermediate regime in the TOF expansion leads to more complex features, including smaller-sized motifs, in the interference patterns. Finally, when accounting for our imaging resolution, it yields a broadening of the structure observed in the TOF images and less visible peaks in the FT (see (i)). We note that our experiments, because of limited N and additional losses, should rather lie in the regime  $2 \le N_D \le 5$ ; see Fig. 1(b).



FIG. S2. Toy model realizations with varying  $\sigma_y/d$ . We use 100 independent draws, with  $N_D = 4$ ,  $d = 2.85 \,\mu\text{m}$ (with 10% noise) and either  $\phi_i = 0$  (a,b,e,f,i,j), or  $\phi_i$  uniformly distributed between 0 and  $2\pi$  (c,d,g,h,k,l). For each realization we also compute the associated mean S. (a–d)  $\sigma_y/d = 0.1$ , yielding  $S = 1.8 \times 10^{-7}$  (e–h) $\sigma_y/d = 0.15$ , matching  $S = 1.7 \times 10^{-4}$  and (i–l)  $\sigma_y/d = 0.25$ , matching S = 0.028. (a,c,e,g,i,k) TOF density profiles and (b,d,f,h,j,l) Corresponding FT analysis of the interference patterns, same color code as Fig. 2.

We then investigate the evolution of the interference patterns and their FT analysis for a varying mean droplet size,  $\sigma_y$ , while keeping their mean distance, d, fixed. This study is particularly relevant recalling that, within the Josephson junction formalism (see main text and corresponding section of this Supplemental Material), the key parameter controlling the tunneling rate between the droplets is set by the ratio  $\sigma_y/d,$  and the link strength parameter that we use to characterize the supersolid regime scales roughly as  $\exp(-(d/2\sigma_y)^2)$ . Thus, in our experiment,  $\sigma_u/d$  is intrinsically expected to decrease with the scattering length (see Fig. 3). Performing a direct estimate of the average droplet link from the initial state of our toy model, we find S = 0.004 for the calculations of Fig. 2(j-o), lying in an expected supersolid regime yet rather close to the supersolid-to-ID transition. Figure S2 investigates the effect of smaller and larger values of  $\sigma_u/d$  (and consequently of S) on the TOF and FT profiles while independently assuming phase coherence or incoherence. Qualitatively, the features remain similar as in Fig. 2(j-o). In the coherent case, side peaks are visible in

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the individual as well as in the mean  $n(k_y)$  (see (a,e,i)) and yield side peaks in the FT profiles, with  $n_{\mathcal{M}} \approx n$  (see (b,f,j)). Increasing (decreasing)  $\sigma/d$  mainly results in a stronger (weaker) signal both in the interference pattern and their FT analysis. Within our toy model, we find that, already for  $\sigma/d = 0.25$ , the signal nearly vanishes; see (i,j). Even if, given the approximations used in our toy model, this exact value may not fully hold for our experimental conditions, we expect a similar trend. It is interesting to keep in mind that this effect may limit our capacity of detecting an underlying supersolid state via matter-wave interference in experiments. In the incoherent case, the effect of decreasing  $\sigma_y/d$  mainly results in a broader shape of the mean density profile, while it remains non-modulated; see (c,g,k). In the FT analysis  $n_{\Phi}$ remains structure-less independently of  $\sigma_y/d$  while the structures in  $n_{\mathcal{M}}$  becomes sharper with decreasing  $\sigma_y/d$ , as in the coherent case: see (d,h,l).



FIG. S3. Toy model realizations allowing noise in the center position. We use 100 independent draws, with  $N_D = 4$ ,  $d = 2.85 \,\mu\text{m}$  (with 10% noise),  $\sigma_y/d = 0.15$  (ad) or  $\sigma_y/d = 0.2$  (e-h), and either  $\phi_i = 0$  (a,b,e,f,i,j), or  $\phi_i$ uniformly distributed between 0 and  $2\pi$  (c,d,g,h,k,l). Center fluctuation are introduced as normal noise around 0 with standard deviation of  $2\mu\text{m}^{-1}$  in situ (a,c,e,g,i,k) TOF density profiles and (b,d,f,h,j,l) corresponding FT analysis of the interference patterns, same color code as Fig. 2.

Finally, we investigate how a possible shot-to-shot noise on the position of the central interference peak could affect our observables of the density modulation and phase coherence. In the experiments, such fluctuations may occur, for instance, because of beam-pointing fluctuations or excitations of the gas. Although we compensate for such effects by recentering the individual images (see Imaging Analysis section), residual effects may remain, in particular due to center misestimation in the mere presence of the interference patterns of interest. To investigate this aspect, we repeat our toy model calculations now including noise in the global droplet array position and using a standard deviation of  $2\,\mu\mathrm{m}$  for two values of  $\sigma_u/d$ ; see Fig.S3. Again, qualitatively the observed features remains similar to our prediction in the main text. The main effect lies in the appearance of a

small discrepancy in the coherent case between  $n_{\Phi}$  and  $n_{\mathcal{M}}$ , while the structure in the incoherent case remains similar. As the center misestimation should be the most severe in the latter case (due to the variability of the interference patterns observed here), our test shows the robustness of our analysis procedure against this issue.

#### IMAGING ANALYSIS: $^{164}\mathrm{Dy}$ AND $^{166}\mathrm{Er}$

The density distributions in momentum space are extracted from the TOF images using the free-expansion expectation. In the Dy case, the thermal component is subtracted from the individual distribution by cutting out the central region of the cloud and performing an isotropic Gaussian fit on the outer region. This subtraction is beneficial because of the large thermal fraction. In the  $^{166}\mathrm{Er}$  case, such a subtraction is on the contrary complicated because of the weak thermal component and this pre-treatment may lead to improper estimation of  $A_{\mathcal{M}}$  and  $A_{\Phi}$  in the later analysis. The obtained momentum density distributions are then recentered and integrated numerically along  $k_z(k_x)$  between  $[-2.0, +2.0] \,\mu\text{m}^{-1} ([-1.28, +1.28] \,\mu\text{m}^{-1})$  to obtain  $n(k_Y)$  $(n(k_y))$  for <sup>164</sup>Dy (<sup>166</sup>Er). The recentering procedure uses the result a single Gauss fit to the TOF images. The fit is performed after convoluting each image with a Gaussian function of width  $0.5\,\mu m$  whose purpose is to reduce the impact of the interference pattern on the center estimation [19].

In order to characterise the system's state, we use the Fourier transform,  $\mathcal{F}[n](y)$  of the single density profile,  $n(k_y)$ . We then compute two average profiles,  $n_{\mathcal{M}}$  and  $n_{\Phi}$ , relying on ensemble average over all measurements under the same experimental conditions; see below for a detailed discussion on  $n_{\mathcal{M}}$  and  $n_{\Phi}$ . In all the measurements reported in this work we use averages over typically 15 to 100 realizations.

To quantify both the existence of a density modulation and global phase coherence on top of this modulation, we fit both  $n_{\mathcal{M}}(y)$  and  $n_{\Phi}(y)$  with a triple-Gaussian function, where one Gaussian accounts for the central peak and the other Gaussians are accounting for the symmetric side peaks. The amplitudes of the latter give  $A_{\mathcal{M}}$  and  $A_{\Phi}$ , respectively. The distance between the side peaks and the central one is allowed to vary between [2.5, 2.7]  $\mu$ m ([2.3, 2.5]  $\mu$ m) in the case of <sup>164</sup>Dy (<sup>166</sup>Er).

#### DETAILS ON THE FOURIER ANALYSIS

In our analysis we rely on two averaged profiles, named  $n_{\mathcal{M}}$  or  $n_{\Phi}$ , to quantify both the density modulation and its phase coherence. Here we detail the meaning of the average performed.

The Fourier transform (FT) of the integrated momentum distributions,  $n(k_y)$ , which reads  $\mathcal{F}[n](y) =$  $|\mathcal{F}[n](y)| \exp(i \arg(\mathcal{F}[n](y)))$  sets the ground for our analysis. As stated in the main text, an in-situ density modulation of wavelength  $y^*$  yields patterns in  $n(k_y)$  and consequently induce peaks at  $y \approx y^*$ , in the FT norm,  $|\mathcal{F}[n](y)|$ , see Fig. 2(g-i) and (m-o). Spatial variations of the phase relation within the above-mentioned density modulation translate into phase shifts of the interference patterns, which are stored in the FT argument at  $y \approx y^*$ ,  $\arg(\mathcal{F}[n](y^*))$ ; see also Ref. [18, 20].

The first average that we use is  $n_{\mathcal{M}}(y) = \langle |\mathcal{F}[n](y)| \rangle$ , i. e. the average of the FT norm of the individual images. As the phase information contained in  $\arg(\mathcal{F}[n](y))$  is discarded from  $n_{\mathcal{M}}$  when taking the norm, the peaks in  $n_{\mathcal{M}}$  probe the mere existence of an insitu density modulation of roughly constant spacing within the different realizations. The second average of interest is  $n_{\Phi}(y) = |\langle \mathcal{F}[n](y) \rangle|$ , i. e. the average of the full FT of the individual images. In contrast to  $n_{\mathcal{M}}$ ,  $n_{\Phi}$  keeps the phase information of the individual realizations contained in  $\arg(\mathcal{F}[n](y^*))$ . Consequently, peaks in  $n_{\Phi}$  indicate that the phase relation is maintained over the density modulation, in a similar way for all realizations. Their presence thus provides information on the global phase coherence of a density-modulated state.

#### **EXPERIMENTAL SEQUENCE:** <sup>164</sup>Dy **AND** <sup>166</sup>Er

 $^{166} Erbium$  - The BEC of  $^{166} {\rm Er}$  is prepared similarly to Refs. [1, 8, 21, 22]. We start from a magneto-optical trap with  $2.4 \times 10^{7}$  <sup>166</sup>Er atoms at a temperature of  $10\mu$ K, spin-polarized in the lowest Zeeman sub-level. In a next step we load about  $3 \times 10^6$  atoms into a crossed optical dipole trap (ODT) operated at 1064 nm. We evaporatively cool the atomic cloud by reducing the power and then increasing the ellipticity of one of the ODT beams. During the whole evaporation a constant magnetic field of  $\tilde{B} = 1.9 \,\text{G} (a_{\text{s}} = 80 \,a_0)$  along z is applied. We typically achieve BEC with  $1.4 \times 10^5$  atoms and a condensed fraction of 70%. In a next step the ODT is reshaped in 300 ms into the final trapping frequencies  $\omega_{x,y,z} = 2\pi \times (227, 31.5, 151)$  Hz. Consecutively, we ramp B linearly to  $0.62\,{\rm G}~(64.5\,a_0)$  in 50 ms and obtain a BEC with  $8.5 \times 10^4$  atoms, which are surrounded by  $3.5 \times 10^4$ thermal atoms. This point marks the start of the ramp to the final  $a_s$ .

 $^{164}Dysprosium$  - For the production of a  $^{164}Dy$  BEC we closely follow the scheme presented in [23]. Starting from a 3 s loading phase of our 5-beam MOT in open-top configuration [24], we overlap a 1064 nm single-beam dipole trap with a  $^{1/e^2}$ -waist of about  $22\,\mu\mathrm{m}$ , for 120 ms. Eventually, we transfer typically  $8\times10^6$  atoms utilizing a time averaging potential technique to increase the spatial overlap with the MOT. After an initial 1.1s evaporative

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cooling phase by lowering the power of the beam, we add a vertically propagating beam, derived from the same laser, with a  $1/e^2$ -waist of about 130  $\mu$ m to form a crossed optical dipole trap for additional confinement. Subsequently, we proceed forced evaporative cooling to reach quantum degeneracy by nearly exponentially decreasing the laser powers in the two dipole-trap beams over 3.6 s. We achieve BECs of  $^{164}$ Dy with typically 10<sup>5</sup> atoms and condensate fractions of about 40%. During the entire evaporation sequence the magnetic field is kept constant at 2.5 G pointing along the vertical (z-) axis.

To be able to condense directly into the supersolid, we modify the dipole trap to condense at a stronger confinement of  $\omega_{x,y,z} = 2\pi \times (225, 37, 134)$  Hz. After a total evaporative cooling duration of 3.1 s, we achieve Bose-Einstein condensation at 2.55 G and reach a state with supersolid properties at 2.43 G, keeping the magnetic field constant throughout the entire evaporation sequence for both cases.

Time of flight and imaging for <sup>166</sup>Er and <sup>164</sup>Dy - In order to probe the momentum distribution of the Dy (Er) gases, we switch off the confining laser beams and let the atoms expand freely for 18 ms (15 ms), while keeping the magnetic field constant. Consecutively the amplitude of B is increased to a fixed amplitude of 5.4 G (0.6 G). In the case of <sup>164</sup>Dy, the magnetic field orientation is rotated in order to point along the imaging axis. This ensures constant imaging conditions for different  $a_{\rm s}$ . After an additional 9 ms (15 ms) we perform a standard absorption imaging.

## TUNING THE SCATTERING LENGTH IN $^{166}\mathrm{Er}$ AND $^{164}\mathrm{Dy}$

 $^{166}Erbium$  - All measurements start with a BEC at 64.5  $a_0$ . In order to probe the BEC-supersolid-ID region, we linearly ramp  $a_s$  to its target value in  $t_{\rm r}=20\,{\rm ms}$  by performing a corresponding ramp in B. Due to a finite time delay of the magnetic field in our experimental setup and the highly precise values of  $a_s$  needed for the experiment, we let the magnetic field stabilize for another 15 ms before  $t_h=0$  starts. By this, we ensure that the residual lowering of  $a_s$  during the entire hold time is  $\lesssim 0.3 a_0$ . In the main text, we always give the  $a_s$  at  $t_h=0$ . Furthermore, we estimate our magnetic field uncertainty to be  $\pm 2.5 \,{\rm mG}$ , leading to a  $\pm 0.2 \,a_0$  uncertainty of  $a_s$  in our experiments.

To choose the best ramping scheme, we have performed experiments varying  $t_{\rm r}$  from 0.5 ms to 60 ms, ramping to a fixed  $a_{\rm s}$  lying in the supersolid regime, and holding for  $t_{\rm h} = 5$  ms after a fixed 15 ms waiting time. We record the evolution of  $A_{\Phi}$  as a function of  $t_{\rm r}$ ; see Fig.S4. When

increasing  $t_r$ , we first observe that  $A_{\Phi}$  increases, up to  $t_r = 20 \,\mathrm{ms}$ , and then  $A_{\Phi}$  gradually decreases. The initial increase can be due to diabatic effects and larger excitation when fast-crossing the phase transition. On the other hand, the slow decrease at longer  $t_r$  can be explained by larger atom loss during the ramp. We then choose  $t_r = 20 \,\mathrm{ms}$  as an optimum value where a supersolid behavior develops and maintains itself over a significant time while the losses are minimal.



FIG. S4. Ramp time effect on the supersolid behavior Measured  $A_{\Phi}$  for various durations of the scattering-length ramp with <sup>166</sup>Er and a final  $a_s = 54.1(2) a_0$ . All measurements include a 15 ms stabilization time after  $t_r$  and are performed with an additional hold of  $t_h = 5$  ms.

<sup>164</sup>Dysprosium - As the value of the background scattering,  $a_{\rm bg}$  length for <sup>164</sup>Dy is still under debate [9, 10, 25], we discuss the experimental settings in terms of magnetic field. Yet, to gain a better understanding of the tunability of  $a_s$  in our experiment, we first perform a Feshbach spectroscopy scan on a BEC at  $T = 60 \,\mathrm{nK}$ . After evaporative cooling at  $B = 2.5 \,\mathrm{G}$ , we jump to B varying from 1 G to 7.5 G and we hold the sample for 100 ms. Finally, we switch off the trap, let the cloud expand for  $26\,ms$  and record the total atom number as a function of B. We then fit the observed loss features with a gaussian fit to obtain the position  $B_{0,i}$  and width  $\Delta B_i$  of the FRs, numbered i. We finally use the standard Feshbach resonance formula to estimate the  $a_s$ -to-B dependence via  $a_{\rm s}(B) = a_{\rm bg} \prod_i (1 - \Delta B_i / (B - B_{0,i}))$ . Here we account for 8 FRs located between 1.2 G and 7.2 G. Depending on the background scattering length  $a_{bg}$ , the overall magnitude of  $a_{\rm s}(B)$  changes. We can get an estimate of  $a_{\rm bg}$ from literature. In Fig. S5, we use the value of  $a_s$  from Ref. [25] obtained at 1.58 G close to the B-region investigated in our experiment,  $a_s = 92(8) a_0$ . By reverting

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the  $a_{\rm s}(B)$  formula, we set  $a_{\rm bg}=87(8)\,a_0$ . For the measurements of Figs. 4-5, we ramp *B* linearly from 2.5 G in 20 ms to a final value ranging from 1.8 to 2.1 G, for which we estimate  $a_{\rm s}$  ranging from 97(9)  $a_0$  to 105(10)  $a_0$ . We calibrate our magnetic field using RF spectroscopy, with a stability of about 2 mG. In the Dy case, we do not apply an additional stabilization time. This is justified because of the more mellow  $a_{\rm s}$ -to-*B* dependence in the *B*-range of interest as well as of the wider  $a_{\rm s}$ -range of the super-oslid regime (see Fig. 1) compared to the Er case. For the measurements of Figs. 6–7, we use two *B*-values, namely 2.43 G and 2.55 G, at which we perform the evaporative cooling scheme. Here we estimate  $a_{\rm s} = 109(10) a_0$  and  $a_{\rm s} = 134(12) a_0$ , respectively.



FIG. S5. Estimated scattering length tuning in <sup>164</sup>Dy Estimated dependence of  $a_{\rm s}$  on *B* for <sup>164</sup>Dy. The FR positions and widths have been extracted from trap-loss spectroscopy measurements, the background scattering length is estimated to  $a_{\rm bg} = 87(8) a_0$ , see text. The blue dashed line gives an error-estimate considering only the errorbar on  $a_{\rm bg}$ from the mere  $a_{\rm s}$  measurement of Ref. [25] and not accounting for uncertainty of the Feshhach scan. For Figs. 4-5, we use *B* between 1.8 G and 2.1 G (red area); for Figs. 6–7, we keep at two constant *B*-values, namely 2.43 G and 2.55 G (red arrows).

#### ATOM LOSSES IN <sup>166</sup>Er AND <sup>164</sup>Dy

As pointed out in the main text, in the time evolution of the quantum gases in both the supersolid and the ID regime, inelastic atom losses play a crucial role. The atom losses are increased in the above mentioned regime as (i) higher densities are required so that a stabilization under quantum fluctuation effects becomes relevant and (ii) the magnetic field may need to be tune close to a FR pole to access the relevant regime of interaction parameters. (i) is at play for all magnetic species but more significant for <sup>166</sup>Er due to the smaller value of  $a_{dd}$ . (ii) is relevant for both <sup>166</sup>Er and <sup>162</sup>Dy but conveniently avoided for <sup>164</sup>Dy thanks to the special short-range properties of this isotope.

To quantify the role of these losses, we report here the evolution of the number of condensed atoms, N, as a function of the hold time in parallel to the phase coherent character of the density modulation observed. We count N by fitting the thermal fraction of each individual image with a two-dimensional Gaussian function. To ensure that only the thermal atoms are fitted, we mask out the central region of the cloud associated with the quantum gas. Afterwards we subtract this fit from the image and perform a numerical integration of the resulting image (so called pixel count) to obtain N.



FIG. S6. atom number and coherence decays in <sup>166</sup>Er Time evolution of N and  $A_{\Phi}$  for <sup>166</sup>Er at different  $a_s$ , including points before  $t_{\rm h} = 0$  ms in the experiment. The corresponding scattering lengths are 53.3(2)  $a_0$  (a,b), 54.0(2)  $a_0$ (c,d), 54.2(2)  $a_0$  (e,f).

 $^{166} Erbium$  - In the Er case, a 15 ms stabilization time is added to ensure that  $a_{\rm s}$  is reached up to  $0.3\,a_0$ . During this time, i.e. for  $t_{\rm h} < 0$ , we suspect that the time-evolution of the cloud properties is mainly dictated by the mere evolution of the scattering length. Therefore, in the main text, we report on the time evolution for  $t_{\rm h} \geq 0$ . We note that because of the narrow  $a_{\rm s}$ -range for the supersolid regime, the long stabilization time for  $a_{\rm s}$  is crucial. However, because of the significant role of the atom losses in our system, in particular for  $^{166}{\rm Er}$ , the early evolution of N and the cloud's properties are intimately connected. Therefore, the early time evolution at  $t_h < 0$  is certainly of high importance for our observations at  $t_h \geq 0$ .

To fully report on this behavior, we show the evolution of N and  $A_{\Phi}$  during both the stabilization and the holding time in Fig. S6 for three different  $a_{\rm s}$  values – either in the ID (a, b) or supersolid regime (c-f). The time evolution shows significant atom loss, prominent already during the stabilization time, and levels off towards a remaining atom number at longer holding times in which we recover small BECs. Simultaneously, in each case reported here, we observe that during the stabilization time  $A_{\Phi}$ increases and a coherent density modulated state grows. TABLE I. Extracted 1/10-lifetime of <sup>166</sup>Er atom number decay for  $t_{\rm h} \ge 0$  and remaining atom number at long holding time for data in Fig. S6.

| $a_s(a_0)$ | $t_N (\mathrm{ms})$ | $N_r(10^4)$ | $t_{\Phi} (\mathrm{ms})$ |
|------------|---------------------|-------------|--------------------------|
| 53.3(2)    | 32(5)               | 1.03(5)     | -                        |
| 54.0(2)    | 51(9)               | 1.29(11)    | 25(6)                    |
| 54.2(2)    | 46(12)              | 1.7(2)      | 32(9)                    |

This density modulation starts to appear at a typical atom number of  $N \gtrsim 6 \times 10^4$  and consecutively decays. For the lower  $a_8 = 53.3(2) a_0$  case, we observe that the coherent state does not survive the  $a_{\rm s}$  stabilization time, and decays faster than the atoms loss; see Fig. S6(a, b). This behavior corresponds to the ID case discussed in the main text. The central point of the present work is to identify a parameter range where the coherence of the density modulated state survives for  $t_{\rm h} > 0$  and its decay time scale is similar to the one of the atom loss. In order to quantify a timescale for the atom number decay, we fit an exponential decay to  $t_{\rm h} \ge 0 \,\mathrm{ms}$ . Here we allow an offset  $N_r$  of the fit, accounting for the BEC recovered at long holding times. In Table I, we report on the typical 1/10-decay times of the atom number, which are up to 50 ms. These values are of the order as the extracted  $t_{\Phi}$ , see Table I and Fig. 5 of the main text. This reveals that in <sup>166</sup>Er the extracted lifetime of the coherent density modulated states are mainly limited by atom loss.

Furthermore we note that the extracted  $N_r$  values for the recovered BECs are smaller than  $2 \times 10^4$ , which is consistent with the BEC region found in the phase diagram of Fig. 1(f).

 $^{164}Dysprosium$  - Differently from the  $^{166}\mathrm{Er}$  case, for  $^{164}\mathrm{Dy},$  we operate in a magnetic-field range in which the three-body collision coefficients are small and only moderate atom losses occur. This enables the observation of an unprecendented long-lived supersolid behavior. To understand the effects limiting the supersolid lifetime, we study the lifetime of the condensed-atom number for different B. We perform this detailed study for the data of Fig. 5 of the main text, which are obtained after preparing a stable BEC and then ramping B to the target value. Fig. S7 shows the parallel evolution of N and  $A_{\Phi}$  for three different magnetic field values 1.8 G, 2.04 G and 2.1 G. Here we observe that, for all B values,  $A_{\Phi}$  seems to decay faster than the atom number. This suggests that the lifetime of the density-modulated state in our  $^{164}\mathrm{Dy}$  experiment is not limited by atom losses. To confirm this observation, we extract the 1/10 lifetimes of both N and  $A_{\Phi}$ ; see Table II. The values confirm our observation and shows an atom number lifetime larger than  $t_{\Phi}$  at least by a factor of  $\approx 5$ . In addition, we find that the ratio  $t_N/t_{\Phi}$ varies, indicating that atom losses are not the only mechanism limiting the lifetime of the supersolid properties in Dy.

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FIG. S7. atom number and coherence decays in <sup>164</sup>Dy Time evolution of N and  $A_{\Phi}$  for <sup>164</sup>Dy at different B for the data of Fig. 5. The corresponding magnetic fields are 1.8 G (a,b), 2.04 G (c,d), 2.1 G (e,f).

TABLE II. Extracted 1/10-lifetime of  $^{164}\mathrm{Dy}$  atom number decay and  $A_{\Phi}$  decay for data in Fig. S7.

| B(G) | $t_N (\mathrm{ms})$ | $t_{\Phi} (\mathrm{ms})$ |
|------|---------------------|--------------------------|
| 1.8  | 300(12)             | 12(5)                    |
| 2.04 | 728(34)             | 152(13)                  |
| 2.1  | 926(36)             | 133(25)                  |

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### BIRTH, LIFE, AND DEATH OF A DIPOLAR SUPERSOLID

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For this publication, I (M.S.) developped a high-resolution imaging system and experimental procedures, performed the measurements together with C.P., L.K. and M.N., lead the data analysis together with M.N., and wrote the manuscript with contributions from all other authors.

PHYSICAL REVIEW LETTERS 126, 233401 (2021) Editors' Suggestion Featured in Physics Birth, Life, and Death of a Dipolar Supersolid Maximilian Sohmen<sup>(0)</sup>,<sup>1,2</sup> Claudia Politi<sup>(0)</sup>,<sup>1,2</sup> Lauritz Klaus<sup>(0)</sup>,<sup>1,2</sup> Lauriane Chomaz<sup>(0)</sup>,<sup>2,†</sup> Manfred J. Mark<sup>(0)</sup>,<sup>1,2</sup> Matthew A. Norcia<sup>®</sup>,<sup>1</sup> and Francesca Ferlaino<sup>®1,2,</sup> <sup>1</sup>Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria Institut für Experimentalphysik, Universität Innsbruck, 6020 Innsbruck, Austria (Received 19 January 2021; accepted 19 April 2021; published 7 June 2021) In the short time since the first observation of supersolid states of ultracold dipolar atoms, substantial progress has been made in understanding the zero-temperature phase diagram and low-energy excitations of these systems. Less is known, however, about their finite-temperature properties, particularly relevant for supersolids formed by cooling through direct evaporation. Here, we explore this realm by characterizing the evaporative formation and subsequent decay of a dipolar supersolid by combining high-resolution intrap imaging with time-of-flight observables. As our atomic system cools toward quantum degeneracy, it first undergoes a transition from thermal gas to a crystalline state with the appearance of periodic density modulation. This is followed by a transition to a supersolid state with the emergence of long-range phase coherence. Further, we explore the role of temperature in the development of the modulated state.

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Supersolid states, which exhibit both global phase coherence and periodic spatial modulation [1–7], have recently been demonstrated and studied in ultracold gases of dipolar atoms [8–10]. These states are typically accessed by starting with an unmodulated Bose-Einstein condensate (BEC) and then quenching the strength of interatomic interactions to a value that favors a density-modulated state. In this production scheme, the superfluidity (or global phase coherence) of the supersolid is inherited from the preexisting condensate. However, a dipolar supersolid state can also be reached by direct evaporation from a thermal gas with fixed interactions, as demonstrated in Ref. [10].

A thermal gas at temperatures well above condensation has neither phase coherence nor modulation, so both must emerge during the evaporative formation process. This leads one to question whether these two features appear simultaneously, or if not, which comes first. Further, because this transition explicitly takes place at finite temperature T, thermal excitations may play an important role in the formation of the supersolid, presenting a challenging situation for theory. Moreover, in the case of a dipolar supersolid, the nonmonotonic dispersion relation and the spontaneous formation of periodic density modulation lead to important new length and energy scales not present in contact-interacting systems, which dramatically modify the evaporative formation process.

While the ground state and dynamics of a zero-temperature dipolar quantum gas can be computed by solving an extended Gross-Pitaevskii equation [8,11–17] [see also Fig. 1(a)], similar treatments are currently lacking for finite temperatures in the supersolid regime. In principle, effects of finite temperature can be taken into account by perturbatively including the thermal population of excited modes.

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Fock-Bogoliubov equations [9,21,22]. In order to accurately describe dynamical processes occurring at temperatures approaching the critical temperature, both coherent excitations and incoherent interactions with the background thermal gas must be accounted for, requiring either more advanced *c*-field [18] or quantum Monte Carlo [23–27] techniques. So far, theories with realistic experimental parameters have not been developed to unveil the finite-temperature dipolar phase diagram and to determine the properties of the thermal-to-supersolid phase transition. In this Letter, we experimentally study the evaporative transition into and out of a supersolid state in a dilute gas of

This can be done either coherently, by adding them in a

single classical field that abides the Gross-Pitaevskii

equation, as in Refs. [18-20], or incoherently, by iteratively

computing mode populations via a set of coupled Hartree-

transition into and out of a supersolid state in a dilute gas of dysprosium atoms. As the atoms cool down to quantum degeneracy, the number of condensed atoms increases, giving birth to the supersolid state. Continued evaporation and collisional loss lead to a reduction of atom number and, eventually, the death of the supersolid. Such an evaporation trajectory, as illustrated in Fig. 1(a), passes through the little-understood finite-temperature portion of the supersolid phase diagram. During the evaporative birth of the supersolid, we discover that the system first establishes strong periodic density modulation of locally coherent atoms and only later acquires long-range phase coherence. When comparing the birth and death of the supersolid, which occur at different temperatures, we observe higher levels of modulation during the birth, suggesting that thermal fluctuations may play an important role in the formation of density modulation.

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FIG. 1. Evaporation trajectory through the finite-temperature phase diagram. (a) At T = 0 (bottom plane), the phase diagram for a gas of dipolar atoms is spanned by the s-wave scattering length  $a_s$  and the condensate atom number  $N_c$ . In an elongated trap, it features a BEC (white) and independent droplet (ID, black) phases, separated in places by a supersolid state (SSS, gray scale). The plotted lightness in the T = 0 phase diagram represents the droplet link strength across the system (cf. Ref. [16]). Away from T = 0, the phase diagram is not known. We explore this region through evaporation into [red, near (i)] and out of [blue, near (ii)] the SSS, along a trajectory represented schematically by the colored arrow. (b) Single-shot image of the optical density (OD) of the sample in trap. Here, a system of four "droplets" within the SSS region is shown, together with its projected density profile. (c) Single-shot matter-wave interference pattern after 35 ms TOF expansion (OD) and the corresponding projected profile. The color scale is truncated for visual clarity. The background clouds of thermal atoms present are not visible in the color scales of (b) or (c); for 35 ms TOF and around 50 nK [as in (c)], the thermal atoms show an approximately isotropic 2D Gaussian distribution of mean width  $\bar{\sigma} \sim 55 \ \mu m$ .

For our experiments, we first prepare an optically trapped gas of approximately 10<sup>5</sup> dysprosium atoms (isotope 164Dy), precooled via forced evaporation to temperatures of several hundred nanokelvin, at which point the gas remains thermal. From here, we can apply further evaporation either by a nearly adiabatic ramp-down of the trap depth ("slow ramp") or by a rapid reduction of the trap depth followed by a hold time at fixed depth ("fast ramp") to further lower the temperature and induce condensation into the supersolid state. The slow ramp protocol yields a higher number of condensed atoms ( $N_c \sim 2 \times 10^4$ ; see next paragraph for definition) and lower shot-to-shot atom number fluctuations, whereas the fast ramp protocol  $(N_c \sim 10^4)$  allows to follow the evolution of the system in a constant trap, disentangling the system dynamics from varying trap parameters. In contrast to protocols based on quenching the interactions in a BEC [8-10], we hold the magnetic field (and hence the contact interaction strength) fixed during the entire evaporation process at 17.92 G, where the system ground state at our  $N_c$  is a supersolid [scattering length ~  $85(5) a_0$ ].

For the present Letter, we have implemented *in-situ* Faraday phase contrast imaging [28,29], which allows us to

probe the in-trap density of our quantum gas at micronscale resolution. During the formation of the densitymodulated state, the translation symmetry is broken along the long (axial) direction of our cigar-shaped trap [30], typically giving rise to a chain of 3-6 density peaks, which we call droplets. These droplets have a spacing of roughly 3 µm, clearly visible in our in-situ images [Fig. 1(b)]. As in our previous works [10,16], we also image the sample after a time-of-flight (TOF) expansion using standard absorption imaging. These TOF images include a spatially broad contribution that we attribute to thermal atoms, whose number  $N_{\rm th}$  and temperature T we estimate by 2D fitting of a Bose-enhanced Gaussian function [31], excluding the cloud center. Surplus atoms at the cloud center (compared to the broad Gaussian) are at least locally coherent, or "(quasi-)condensed" in the sense of Refs. [32-34]. With the total number of atoms N measured by pixel count, we define  $N_c = N - N_{\text{th}}$  to be the number of these (at least locally) coherent atoms. During TOF, matter-wave interference between the expanding droplets gives rise to a characteristic interference pattern [Fig. 1(c)]. The high contrast of the interference pattern is visible in single TOF images and indicates that each individual droplet is by itself a phase-coherent many-body object. The stability of the interference fringes within the envelope over multiple experimental realizations encodes the degree of phase coherence between droplets (cf. Refs. [10,16] and discussion below). The combination of in situ and TOF diagnostics provides complementary information, allowing us to measure both density modulation and its spatial extent (number of droplets), as well as phase coherence.

Figure 2 shows the birth of the supersolid. Starting from a thermal sample, we apply the fast ramp (225 ms) evaporation protocol to the desired final trap depth, too fast for the cloud to follow adiabatically and intermediately resulting in a nonthermalized, noncondensed sample. Simply holding the sample at constant trap depth for a time  $t_h$ , collisions and plain evaporation lead to thermalization and cooling. In Fig. 2(a), we plot the average axial *in-situ* density profile [cf. Fig. 1(b)] versus  $t_h$ , for about 20 images per time step without any image recentering. At early  $t_h$  the atoms are primarily thermal and show up as a broad, low-density background in our images. For  $t_h \lesssim 150$  ms, inspection of single-shot images reveals an increasing, though substantially fluctuating, number of droplets appearing out of the thermal cloud. After this time, the droplet number stabilizes to its final value. We observe that the droplet formation happens on the same timescale as the equilibration of  $N_c$  and T (see Supplemental Material [35]). This timescale is set by the rate of evaporation, which in turn depends on the thermalization rate and hence on the elastic collision rate  $1/\tau_{el}$ . For our experimental parameters, we estimate for two thermal atoms  $\tau_{el} \approx 3 \text{ ms} \ (\tau_{el} \approx 10 \text{ ms})$ before (after) our last evaporation ramp. Once the droplets have formed, other timescales might be relevant in



FIG. 2. Growth and spread of density modulation during evaporation. (a) Averaged in-situ density profiles (no recentering, approximately 20 shots per time step) along the long trap axis as a function of hold time  $t_h$  after the fast ramp reduction of trap depth (see main text). (b) The density correlator C'(d) (solid black line) is fitted by a cosine-modulated Gaussian function (dashed red line) to extract the correlation length L. Gray regions are strongly influenced by imaging noise and excluded from fits. Correlators are displayed for  $t_h = 50$  ms (upper) and  $t_h = 300$  ms (lower). (c) Density-density correlation length L versus  $N_c$ , for the same time steps shown in (a). Horizontal error bars are the standard deviation over repetitive shots, vertical error bars reflect the correlator fit uncertainty, red points correspond to the correlators of (b). The dashed line indicates the simple atom number scaling of the Thomas-Fermi radius of a harmonically trapped BEC,  $\propto N_c^{1/5}$ .

determining the equilibration rate of their relative positions and phases; the details of this possibility remain an open question [16].

To better quantify the growth of the modulated state, we consider the density-density correlator C'(d) for the *in-situ* density profiles over distances d [35]. We find that C'(d) is well described by a cosine-modulated Gaussian and define the density correlation length L [Fig. 2(b)] as its fitted width. This method provides a way to determine the extent over which density modulation has formed. Figure 2(c) shows L for the dataset of Fig. 2(a) versus the number of coherent atoms  $N_c$ , which we extract from TOF absorption images in separate experimental trials with identical parameters. Interestingly, despite the strongly modulated

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structure of the supersolid state, the density correlation length L closely follows a scaling  $\propto N_c^{1/5}$ , just as the Thomas-Fermi radius of a harmonically trapped BEC, suggesting a dominant role of interactions over kinetic energy.

While *in-situ* images provide information about density modulation (diagonal long-range order), they do not carry direct information about phase coherence (off-diagonal long-range order), either within or between droplets. For this, we use TOF imaging and address the question of whether the formation of density modulation precedes global (i.e., interdroplet) phase coherence during the evaporative formation of the supersolid, or the other way round.

For this study, we perform a slow (500 ms) final forced evaporation ramp of constant slope that is nearly adiabatic with respect to  $N_c$  and T (though not necessarily with respect to excitations of droplet positions and phase) and terminate the ramp at selected crop times  $t_c$  [38]. After  $t_c$ , we immediately release the atoms and perform TOF imaging. Figure 3(a) shows the observed evolution of the total (N) and (quasi-)condensed  $(N_c)$  atom number, as well as the sample temperature (T) versus  $t_c$ . We expand on the observed evolution by measuring coherence properties. Following Refs. [10,16], for each measurement i we extract a rescaled complex phasor  $P_i = \rho_i \exp(-i\Phi_i)$ , i.e., the Fourier component corresponding to the modulation wavelength in the TOF interference profile (see Supplemental Material [35]). For systems with a small number of droplets (but at least 2), the magnitude of the phasor  $\rho_i$  encodes the modulation strength and also the (local) degree of coherence within each of the individual droplets. Meanwhile, the phase  $\Phi_i$  depends primarily on the relative phase between the droplets (cf. [39]).

We plot the phasors for different evaporation times on the polar plane in Fig. 3(b), where two effects become apparent. First, the modulus of the phasors grows during the evaporation, indicating that the degree of modulation increases. Second, the distribution of phases  $\Phi_i$  is initially uniform and then narrows down over  $t_c$ . To determine the time sequence of these two effects, we calculate the incoherent and coherent amplitude means  $A_M = \langle |P_i| \rangle_i$ , encoding modulation strength and local phase coherence, and  $A_{\Phi} = |\langle P_i \rangle_i|$ , encoding the degree of global phase coherence across the system [10,16]. Plotting  $A_M$  and  $A_{\Phi}$ against  $t_c$  [Fig. 3(c)], we notice a time lag of around 40 ms between the increase of  $A_M$  and  $A_{\Phi}$ , indicating that during evaporation into a supersolid the translational and the phase symmetry are not broken simultaneously [40]. Rather, density modulation and local phase coherence appear before global phase coherence, consistent with predictions from Monte Carlo simulations (cf., e.g., Ref. [27]). A similar effect is observed in the fast ramp protocol [35].

This observation suggests the transient formation of a quasicondensate crystal—a state with local but not



FIG. 3. Development of modulation and coherence while evaporating into the supersolid state. (a) Sample temperature *T* (left ordinate, bullets), total (*N*, right ordinate, dashed red line), and coherent atom number ( $N_c$ , solid red line) as a function of the ramp crop time  $t_c$ . The shadings reflect the respective confidence intervals. (b) The phasors  $P_i$  (black dots), representing the magnitude and phase coherence of modulation for selected  $t_c$  (dotted lines; same radial scale for all polar plots). The red shading reflects mean and variance of the distribution. (c) Evolution of the Fourier amplitude means  $A_M$  (filled markers) and  $A_{\Phi}$  (open markers).

long-range coherence [32-34], whose increased compressibility relative to a thermal gas allows for the formation of density modulation [41]-prior to the formation of a supersolid with phase coherence between droplets. The lack of global phase coherence could be attributed to a Kibble-Zurek-type mechanism [42], in which different regions of the sample condense independently, to excitation of modes involving the motion or phase of the droplets during the evaporation process, or to the thermal population of collective modes (which reduce long-range coherence) at finite temperature. As the evaporation process does not allow independent control of temperature and condensation rate without also changing density or trap geometry, we cannot reliably determine the relative importance of these effects (or others) from the experiment. Dedicated theoretical studies at finite temperature will thus be needed to elucidate the impact of these types of processes and to understand the exact formation process.

After the birth of the supersolid state, both density modulation and global phase coherence persist for remarkably long times, exceeding 1 s. Figure 4 shows the evolution of the coherent atom number  $N_c$  and temperature



FIG. 4. Life cycle of a supersolid state. Density modulation *M* (from *in-situ* images) during the evaporation process (left ordinate, bullets; the vertical error bars reflect the propagated uncertainty returned by the fitting routine). The sample temperature decreases during the hold time  $t_h$  and is encoded by the color filling.  $N_c$  (from TOF images) is the number of coherent atoms over  $t_h$  (right ordinate, red line; the light red shading reflects the measurement standard deviation). At two times where  $N_c \sim 1.1 \times 10^4$  (vertical dashed lines), but at which the atoms have different temperatures, *M* differs substantially. The corresponding averaged *in-situ* images below confirm a higher level of modulation at earlier  $t_h$ . Inset: the observed modulation *M* plotted versus  $N_c$ .

*T* at long hold times under conditions similar to Fig. 2—the same fast ramp followed immediately by hold time  $t_h$ . Evaporative cooling first increases the coherent atom number until, at long  $t_h \ge 1$  s, atom losses become dominant and lead to a continuous decrease of  $N_c$ , eventually leading to the disappearance of the modulated state. However, this death of the supersolid is not a mere time reversal of the birth.  $N_c$  decreases, i.e., evolves in the opposite direction, but more slowly and at lower temperature than for the birth. Furthermore, phase coherence appears to outlive modulation and to be maintained until the very end [35]. Thus, a comparison between the birth and death process provides us with important clues to the impact of temperature on the supersolid.

We contrast the birth and death of the supersolid in Fig. 4 by also plotting the observed *in-situ* density modulation M, which is calculated by Fourier transforming the *in-situ* density profiles and normalizing the Fourier component corresponding to the modulation wavelength to the zero-frequency Fourier component. By comparing M between

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times that have similar  $N_c$  during the birth and the death of the supersolid, respectively, we find that the degree of modulation is higher during the birth of the supersolid than during the death. Because the sample is hotter at shorter hold times, this suggests that the observed modulation is increased at higher temperature, perhaps due to thermal population of collective modes or due to finite-temperature modifications to the dispersion relation [43], as predicted in Ref. [22]. Again, further development of finite-temperature theory will be needed to conclusively determine the importance of such effects.

The role of finite temperature in the formation of modulation, as well as the mechanism by which phase variations across the modulated state arise and then ultimately disappear, represent important future directions for theoretical investigations of dipolar supersolids away from the relatively well understood T = 0 limit. Experimentally, it would be of great interest to study the evaporative formation process in a larger and more uniform system, where distinct domains may be observed to form and a broader separation of length scales may be explored in correlation measurements. Such measurements, along with improved finite-temperature theory, could enable more precise statements as to the nature of the supersolid phase transition away from zero temperature.

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#### Supplementary Material: Birth, life, and death of a dipolar supersolid

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#### CALCULATION OF DENSITY-DENSITY CORRELATOR

We define our correlator as

$$C(d) = \left\langle \int n(x) n(x+d) \, \mathrm{d}x \right\rangle,\tag{1}$$

where n(x) is the projected density at position x along our cigar-shaped trap, and the expectation value  $\langle \dots \rangle$  is calculated over different runs of the experiment. In practice, we follow a standard procedure (e.g., Ref. [1]) and calculate the correlator by computing the square of the Fourier transform of each image to obtain its power spectral density, then Fourier transform again to obtain its autocorrelation function. The autocorrelation functions for the different images in the sample are then averaged to obtain C(d). Note that we do not normalize this as is typical for a noise correlator, as we are interested in the structure of the density profile and not specifically in its fluctuations. To extract the correlation length, we first subtract off a slowly varying background that represents the envelope of our density profile from C(d) to obtain C'(d), shown in Fig. 2b of the main text. We then fit the product of a Gaussian and a cosine with spatial frequency  $k_m = 2\pi/x_m$  corresponding to the in-trap modulation wavelength  $x_m$ , i.e.  $\cos(k_m x) \exp(-x^2/2L^2)$ , and define the correlation length as L.

#### CALCULATION OF COHERENCE QUANTITIES

As described in the main text, we evaluate the coherence of our droplet array by imaging the sample after TOF expansion and Fourier transformation ( $\mathcal{F}$ ) of the projected density profile n(x') (cf. Fig. 1c in main text), where in-situ distances x and the corresponding transforms are denoted as  $x \xrightarrow{\text{TOF}} x' \xrightarrow{\mathcal{F}} x''$  [2]. For each experimental repetition i this yields a phasor

$$\tilde{P}_{i}(x'') = \mathcal{F}\{n(x')\}_{x''}.$$
(2)

We can calculate the incoherent and coherent means of the Fourier amplitudes over the experimental repetitions *i*, writing

$$\tilde{A}_M(x'') = \langle |\tilde{P}_i(x'')| \rangle_i$$
 and  $\tilde{A}_{\Phi}(x'') = |\langle \tilde{P}_i(x'') \rangle_i|,$  (3)

respectively.

The quantities  $A_M$  and  $A_{\Phi}$  from the main text are closely connected to  $\tilde{A}_M(x'')$  and  $\tilde{A}_{\Phi}(x'')$ . To disentangle the spectral amplitude from the coherent atom number (i. e., the area under the density profile), we calculate the rescaled phasors

$$P_{i}(x'') = \frac{\tilde{P}_{i}(x'')}{\int |\tilde{P}_{i}(x'')| \mathrm{d}x''}$$
(4)

mentioned in the main text. The amplitude means corresponding to the in-trap modulation at wavelength  $x_m$ are then given by

 $A_M = \langle |P_i(x_m'')| \rangle_i \quad \text{and} \quad A_\Phi = |\langle P_i(x_m'')\rangle_i|.$ (5)

#### SUPPLEMENTARY DATA FOR FIG. 2

The data of Fig. 2 of the main text is obtained from in-situ images of samples created via the 'fast ramp' evaporation procedure. From corresponding TOF images, taken after the data of Fig. 2, we can study the time evolution of  $\tilde{A}_M(x'')$  and  $\tilde{A}_{\Phi}(x'')$  over the hold time  $t_h$ . After about a hundred milliseconds a sidepeak has developped in  $\tilde{A}_M(x'')$ , corresponding to the in-trap density modulation at  $x_m \sim 3.5 \,\mu\text{m}$  wavelength. A corresponding peak develops in  $\tilde{A}_{\Phi}(x'')$ , signalling growing coherence between the droplets. In Fig. S1 we plot a direct comparison of the rescaled Fourier amplitude means,  $A_M$ and  $A_{\Phi}$ , calculated at  $x''_m = 3.5 \,\mu\text{m}$ . We see that both  $A_M$  and  $A_{\Phi}$  increase with  $t_h$  and the increase of modulation strength starts before the development of phase coherence.

For reference, we plot in Fig. S2 the evolution of the total (N) and coherent  $(N_c)$  atom number for the data set of Fig. 2 of the main text, obtained using the 'fast ramp' evaporation protocol.

#### SUPPLEMENTARY DATA FOR FIG. 4

Fig. 4 of the main text shows the death of the supersolid over long hold times  $t_h$ . Here we compare in Fig. S3 the evolution of  $A_M$  and  $A_{\Phi}$ , calculated at the sidepeak



FIG. S1. Development of coherence after the 'fast **ramp' evaporation**. Evolution of the means  $A_M$  and  $A_{\Phi}$  from the TOF profiles during the hold time  $t_h$ . The gray shading marks the region where due to low overall signal the rescaling of the phasors (Eq. 4) is dominated by noise.



FIG. S2. Atom number and temperature after the 'fast ramp' evaporation. a. Evolution of total (N, dashed) and condensed atom number  $(N_c, \text{solid line})$ . b. Evolution of the temperature, as extracted by a Bose-enhanced Gaussian fit to the background cloud of thermal atoms [3].

in  $\tilde{A}_M$  at  $x'' = 3.5 \,\mu\text{m}$ , during the death of the supersolid. We start our discussion looking at  $A_M$ , the measure for in-trap modulation.  $A_M$  stays roughly constant for around 600 ms before it starts to decay. At above  $\sim 1.1 \text{ s}$  (gray shading in Fig. S3), the sidepeak in  $\tilde{A}_M$ around  $x'' = 3.5 \,\mu\text{m}$  has vanished. However,  $A_M$  does not go straight to zero, since when the modulation disappears, the fundamental peak (around x'' = 0) broadens to  $x'' > 3.5 \,\mu\text{m}$  because the condensate size becomes comparable to the (former) droplet spacing. From this point onwards,  $A_M$  cannot be used anymore as a measure for modulation. Recall that in the in-situ analysis (Fig. 4 of the main text) a very similar behaviour was observed, with maximal modulation until  $\sim 600 \,\text{ms}$  and modulation having disappeared by  $\sim 1.1 \,\text{s}$ .

Now turning to  $A_{\Phi}$ , we note that over the full du-

ration of this process  $A_M$  and  $A_{\Phi}$  (which is bounded by  $A_M$ ) evolve closely together, suggesting that coherence is maintained in the sample throughout the life and death.



FIG. S3. Coherence properties during the death of the supersolid. Evolution of the means  $A_M$  and  $A_{\Phi}$  from the TOF profiles during  $t_h$  after a 'slow ramp' evaporation. The gray shading marks the region from when on the sidepeaks in  $A_M$  corresponding to in-trap modulation have disappeared and  $A_M$  is not a good measure for modulation anymore.

#### EVAPORATION RAMPS

In the experiment, once the atoms are loaded into our crossed optical dipole trap (ODT), we perform a nearexponential evaporation ramp of trap power, approximated by piecewise linear ramp sections. The trap frequencies after the penultimate ramp are around  $\omega'_{x,y,z} =$  $2\pi \times (39, 178, 174) \, \mathrm{s}^{-1}$ , where we typically have around  $N = 3 \times 10^5$  atoms at around 200 nK. From here, we ramp the power of the ODT linearly down to the final value, giving around  $\omega_{x,y,z} = 2\pi \times (36, 88, 141) \, \mathrm{s}^{-1}$ . This procedure yields the atom numbers and temperatures presented in Figs. 3–4 of the main text and Fig. S2.

#### IMAGING SPECIFICATIONS

The images shown in this work have been recorded using a new imaging system recently installed in our experiment. The direction of view of the new system is vertical (counter-directed to gravity).

Images from our imaging along the horizontal direction (as in our earlier works, see, e.g., Refs [4, 5]), in contrast, suffer from the fact that the line of sight is at  $45^{\circ}$  with respect to the axis connecting the droplets, leading to a small apparent fringe spacing and to the interference peaks partially hiding each other; additionally the interference peaks do not lie in a single focus plane. These drawbacks were eliminated with the vertical imaging setup, which is why the images are much clearer to interpret now.

The fundamental resolution of this imaging system, applicable to in-trap images and characterised by the

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Rayleigh criterion, has been measured offline to be approximately 700 nm. We report micron-scale resolution as a conservative claim that accounts for possible alignment imperfections in the finally installed condition, and a reduction of the imaging aperture to increase depth of field. The pixel size of our camera is smaller than the imaging resolution, corresponding to approximately 400 nm at the location of the atoms. Additionally, the images displayed have been supersampled to allow them to be rotated while maintaining resolution.

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# PHASE COHERENCE IN OUT-OF-EQUILIBRIUM SUPERSOLID STATES OF ULTRACOLD DIPOLAR ATOMS

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For this publication, I (M.S.) contributed to works on the experiment, analysed the experimental data with contributions from C. P., performed and analysed simulations based on an extended Gross-Pitaevskii equation together with C.P., L.C., M.J.M. and G.N., and wrote parts of the manuscript with contributions from all other authors. The numerical simulations of Josephson-junction arrays have been performed by G.M. and T.G.

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# Phase coherence in out-of-equilibrium supersolid states of ultracold dipolar atoms

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A supersolid is a counterintuitive phase of matter that combines the global phase coherence of a superfluid with a crystal-like self-modulation in space. Recently, such states have been experimentally realized using dipolar quantum gases. Here we investigate the response of a dipolar supersolid to an interaction quench that shatters the global phase coherence. We identify a parameter regime in which this out-of-equilibrium state rephases, indicating superfluid flow across the sample as well as an efficient dissipation mechanism. We find a crossover to a regime where the tendency to rephase gradually decreases until the system relaxes into an incoherent droplet array. Although a dipolar supersolid is, by its nature, 'soft', we capture the essential behaviour of the de- and rephasing process within a rigid Josephson junction array model. Yet, both experiment and simulation indicate that the interaction quench causes substantial collective mode excitations that connect to phonons in solids and affect the phase dynamics.

The notion of phase coherence lies at the foundation of quantum physics. It is considered a key property in understanding many-body quantum phenomena, ranging from superfluidity and the Josephson effect to the more applied examples of matterwave interference, atom lasing processes and quantum transport in mesoand macroscopic systems<sup>1,2</sup>. Although phase-coherent states are well studied at equilibrium, understanding their out-of-equilibrium dynamics remains an open problem at the forefront of statistical and quantum physics, especially when interactions are present<sup>3</sup>.

Rephasing dynamics of an initially incoherent many-body quantum system requires, first, the system to be conducting such that different parts can exchange energy and particles, and second, an efficient mechanism to dissipate the phase excitations. As for the first requirement, a famous example illustrating the inhibition of thermalization is many-body localization<sup>4</sup>. The second ingredient dissipation—is more subtle and multifaceted, relating, for instance, to the growth of thermal correlations in isolated systems<sup>5</sup>, complex interaction-mediated dynamics<sup>6,7</sup> or the exponential growth of unstable modes and topological defects in connection with the Kibble–Zurek mechanism<sup>6-10</sup>.

The interplay among coherence, self-localization and relaxation dynamics is an intriguing problem. In this respect, the recently discovered<sup>11–13</sup> supersolid states in dipolar quantum gases can potentially provide a new twist in studying non-equilibrium quantum phenomena, about which very little is known so far. A supersolid combines phase coherence and periodic localization in space, properties corresponding to the spontaneous and simultaneous breaking of both gauge and translational symmetry. Intuitively, a supersolid can be viewed as a fully coherent state, which self-establishes compressible density modulation. In this Article, we explore the evolution of a supersolid of ultracold dysprosium (Dy) atoms when brought out of equilibrium after an interaction quench that destroys its global phase coherence. Due to the dynamic formation of the supersolid, an interesting question is whether its phase dynamics are similar to or different from comparable rigid structures, such as a Bose–Einstein condensate (BEC) spliced in an optical lattice<sup>14–16</sup>, or if new phenomena can manifest.

In a dipolar supersolid, the particle self-arrangement in space is largely dictated by the many-body interactions17-22 and can be modified by either tuning the interatomic potentials or changing the atom number (N) in the system. Figure 1a shows the phase diagram of a cigar-shaped quantum gas of bosonic Dy atoms with transverse dipole orientation. It is constructed from the ground-state wavefunctions obtained by numerically solving the extended Gross-Pitaevskii equation (eGPE)<sup>11,13,21</sup> (Methods). Three distinct quantum phases can be accessed by changing N or the s-wave scattering length  $a_s$ , which parametrizes the contact interaction. For a given N and large enough  $a_s$ , the ground state of the system is a non-modulated dipolar BEC (grey region). By lowering a, the influence of the dipolar interaction increases. When reaching a critical value of a<sub>s</sub>, the system undergoes a phase transition to a supersolid phase (SSP). Here density modulation at a wavelength close to the roton excitation23,24 appears in the ground-state density profile (red region). By further lowering  $a_s$ , the system evolves into an array of insulating droplets (IDs) with an exponentially vanishing density link between them (blue region).

Our eGPE calculations, following a standard non-stochastic approach, are inherently phase coherent and thus cannot capture uncorrelated local phases. However, recent experiments have shown a connection between the strength of the density modulation and the coherence properties of the system, revealing a clear difference between the SSP and ID phase<sup>11–13,25</sup>. In the SSP, the whole system shares a global phase. In contrast, in the ID case, any fluctuation or excitation can lead to a locally different evolution of the phase. The absence of particle tunnelling between the droplets leads to dephasing of the system.

By performing interaction quenches and moving across the phase diagram, one can create random phase excitations ('phase scrambling') and thus distinguish between the different coherence characters of the ID and SSP, as shown in Fig. 1b. Our experimental

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**Fig. 1** [Phase diagram, experimental sequence and starting condition. **a**, Ground-state phase diagram for our cigar-shaped trapped <sup>164</sup>Dy gas. The colour map encodes the density modulation via the link strength  $\mathscr{L} = 1 - (n_{max} - n_{min})/(n_{max} + n_{min})$ , where  $n_{max} (n_{min})$  is the density maximum (minimum) in the central region of the calculated in situ density distribution.  $\mathscr{L}$  equals unity for a non-modulated state and zero for a fully modulated state. The grey colour indicates a non-modulated BEC, whereas the red and blue regions correspond to the SSP and ID phase, respectively. The insets show the density profiles along the weak axis for the different phases. **b**, Illustration of the phase-scrambling sequence: starting from an SSP (left) prepared at 2.43 G ( $a_z = 87.9a_0$ ), we reduce  $a_s$  to enter the ID regime (middle). During  $t_s$  the phases of the droplets evolve independently, leading to phase scrambling between the droplets. Eventually, we jump  $a_s$  back into the supersolid regime (right), where the time evolution of the global phase cherence is studied. **c**, **d**, Amplitudes  $A_{M}$  and  $A_{\phi}$  (**d**) for our initial SSP (left panel in **b**) when simply held for time  $t_{M}$ . Each data point is derived from  $q \in [80, 90]$  individual experimental realizations. The error bars (partly covered by plot symbols) are the la confidence intervals from BCA bootstrapping (Methods)<sup>39</sup>. **e**, **f**, Polar scatter plot for  $P_i$  (**e**) and histogram of the probability density function (PDF) (**f**) for  $\Phi_i$  at  $t_h = 100$  ms.

protocol starts by preparing a supersolid state by evaporative cooling from a thermal sample (Fig. 1b, left) (ref. <sup>13</sup>). We then decrease  $a_s$  to enter into the ID regime (Fig. 1b, middle). Here the system ground state is an array of IDs, each with a phase that is expected to evolve independently in time. After these phases have become fully uncorrelated, we jump  $a_s$  back to its initial value (Fig. 1b, right). We then study the time evolution of the out-of-equilibrium system. We measure phase coherence and density modulation, whose coexistence is a hallmark of supersolidity, using a matterwave interference technique (Methods and refs. 11,12 <sup>-29</sup>). In brief, for each experimental realization i, we take an absorption image after a time-of-flight (TOF) expansion. The recorded image exhibits an interference pattern if in-trap density modulation is present. Via Fourier transform, we extract the phasor  $P_i = \rho_i e^{-i\Phi_i}$  with amplitude  $\rho_i$  and phase  $\Phi_i$  at the spatial frequency of the interference pattern (cf. Extended Data Fig. 1a). Averaging over an ensemble of q realizations, the mean of the phasor amplitudes,  $A_{\rm M} = \langle |P_i| \rangle$ , characterizes the degree of density modulation, whereas the amplitude of the complex mean,  $A_{\phi} = |\langle P_i \rangle|$ , contains information about the global phase coherence. As an additional measure of coherence, we calculate the circular variance of the

phase 
$$\Delta \Phi = 1 - \frac{1}{q} \sqrt{\left(\sum_{i=1}^{q} \cos(\Phi_i)\right)^2 + \left(\sum_{i=1}^{q} \sin(\Phi_i)\right)^2}$$
 (ref. <sup>30</sup>)

For a perfect supersolid (resp. ID) state and in the limit  $q \rightarrow \infty$ , one expects  $A_{\phi} = A_{\rm M} > 0$  (respectively  $A_{\phi} = 0$ ,  $A_{\rm M} > 0$ ) and  $\Delta \Phi = 0$ (respectively  $\Delta \Phi = 1$ ). In the following, we describe and characterize each step of our phase-scramble-and-rephase protocol (Fig. 1b).

We initially prepare the supersolid state (Fig. 1b, left) of <sup>164</sup>Dy atoms<sup>13</sup> (N=1.4×10<sup>4</sup>) in an axially elongated optical dipole trap (ODT) with the final harmonic frequencies  $\omega_{xyz}$ =2 $\pi$ ×(225, 37, 165)s<sup>-1</sup>. From our interferometric analysis, we see that this initial state is long lived and has a high degree of phase coherence

(Fig. 1c–f). The characteristic quantities of  $A_{\phi}$  and  $A_{\rm M}$  (Fig. 1c) and  $\Delta \Phi$  (Fig. 1d) are constant over hold times  $t_{\rm h}$  up to 100 ms. The small mean value  $\langle \Delta \Phi \rangle = 0.142(8)$  (the value in parentheses gives the standard error of the last digit) over the entire range of  $t_{\rm h}$  reveals a constantly narrow spread in the phase distribution, as shown for  $t_{\rm h} = 100$  ms in the polar plot (Fig. 1e) of the phasors  $P_i$  and the corresponding histogram (Fig. 1f) for  $\Phi_i$ .

In the next step of our protocol (Fig. 1b, middle), namely, the phase-scrambling excitation, we tune  $a_s$  via magnetic Feshbach resonances by varying the external magnetic field *B* (Methods and Extended Data Fig. 2). From the initial supersolid state (~87.9 $a_0$ , where  $a_0$  denotes the Bohr radius), we transfer the system into the ID regime (~76.9 $a_0$ ) using a 20 ms linear *B*-field ramp. Here the atoms are expected to spatially arrange in an array of almost isolated droplets with exponentially small particle tunnelling between them. We then let the system evolve for a variable scrambling time  $t_s$ .

Figure 2 shows the evolution of  $\Delta \Phi$ ,  $A_{\phi}$  and  $A_{\rm M}$  with  $t_{\rm s}$ . After completion of the *B*-field ramp,  $\Delta \Phi$  initially keeps rapidly increasing for 20 ms and then slowly saturates to a large  $\Delta \Phi$  value. Here the droplets develop uncorrelated phases, as illustrated by the polar plot of  $P_i$  at  $t_{\rm s} = 100$  ms (Fig. 2a, inset). We extract a saturation value of  $\langle \Delta \Phi \rangle_{\rm sat} = 0.92(1)$  (simple mean for  $t_{\rm s} > 50$  ms). We highlight that  $\Delta \Phi$  is not expected to reach unity because of the finite sample size in the experiment ( $q \in [90, 100]$  repetitions; slight variations are due to a post-selection by atom number). It is evident that the measured  $\langle \Delta \Phi \rangle_{\rm sat}$  agrees with the expectation for a sample with the same q and uniformly random phases (grey shading in Fig. 2a, Extended Data Fig. 1b and Methods). As  $\Delta \Phi$  increases and the global phase coherence is lost,  $A_{\phi}$  decreases quickly towards zero (Fig. 2b), whereas the density modulation persists as evidenced by  $A_{\rm M}$  remaining large.



**Fig. 2** | **Phase scrambling. a**,  $\Delta \Phi$  as a function of  $t_s$  when ramping from the SSP to the ID phase (at 1.65 G;  $a_s = 76.9a_0$ ) within 20 ms (similar results are obtained with a fast quench of -1 ms). Each point is derived from  $q \in [90, 100]$  experimental realizations. The error bars are the 1 $\sigma$  confidence intervals from BCA bootstrapping (Methods)<sup>39</sup>. The grey-shaded area indicates the theoretically expected 1 $\sigma$  confidence interval for  $\Delta \Phi$  with same q and uniformly random phases. The inset shows the polar scatter plot of the phasors  $P_i$  at  $t_s = 100$  ms. The light-blue line shows  $\Delta \Phi$  for dephasing in a four-grain JJA model using q = 100 and averaging over four simulation runs (main text and Methods). For the simulation, J is ramped (light-blue circles) and  $A_{\mu}$  (dark-blue diamonds).

The interaction quench into the ID phase is a robust method to create phase-scrambling excitations. It is natural to ask how the relative phases between the individual droplets evolve once the density links between them are restored and whether the system relaxes into an equilibrium supersolid state. To address this question, we perform the last stage of our protocol (Fig. 1b, right). After a scrambling time of  $t_s = 20$  ms, we couple the droplets back together by jumping the *B* field and thus quenching  $a_s$  to  $87.9a_0$ , where the system ground state is a supersolid. However, after the quench, our state is out of equilibrium in terms of both phase and density distribution: since the density-modulated states in dipolar gases are self-assembled, they can deform after a sudden change in the many-body interactions<sup>31-33</sup>. As shown in Fig. 3a, we first observe a rapid reduction in  $\Delta \Phi$  and then slower dynamics towards an equilibrium value with  $\Delta \Phi$  reaching  $\langle \Delta \Phi \rangle_{sat} = 0.13(2)$  (simple mean for  $t_{\rm h} > 50 \, {\rm ms}$ ). Simultaneously,  $A_{\phi}$  approaches  $A_{\rm M}$  on the same timescale, whereas AM remains nearly constant. These observations show that the system efficiently rephases by dissipating the phase excitation.

Our system of multiple superfluid droplets with individual phases, interconnected via weak links, is reminiscent of a Josephson junction array  $(JJA)^{34}$ . Motivated by this analogy, we investigate whether a simple JJA model can adequately describe the observed phase dynamics. This is a non-trivial question, since in contrast to a rigid JJA, our droplet array is 'soft' in the sense that the droplet shapes and their distances change with  $a_s$ . We construct our model from a one-dimensional (1D) array of four coupled grains. For each grain, the number of particles is  $N_j$  and phase is  $\theta_j$ . The Hamiltonian of this system is

$$H = \sum_{j=1}^{4} \frac{(N_j - \overline{N_j})^2}{2C_j} - \sum_{j=1}^{3} J_j \cos(\theta_{j+1} - \theta_j), \quad (1)$$

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**Fig. 3** | **Rephasing. a**,  $\Delta \Phi$  as a function of  $t_h$  after  $t_s = 20$  ms scrambling in the ID phase and a jump back to the SSP regime at 2.43 G ( $a_s = 87.9a_0$ ). For each point,  $q \in [66, 74]$ . The error bars are the  $1\sigma$  confidence intervals from BCA bootstrapping. The inset shows the polar scatter plot of the phasors  $P_i$ at  $t_h = 100$  ms. The light-red line shows  $\Delta \Phi$  for rephasing in a four-grain JJA model using q = 100 and averaging over four simulation runs (main text and Methods). For the simulation, we start with a random configuration of grain phases and let the system rephase with J = 6,000 Hz and  $\eta = 60$ . **b**, The corresponding  $A_{\Phi}$  (light-red circles) and  $A_{M}$  (dark-red diamonds).

where the overline denotes an ensemble average. The first term is the 'charging' energy of the droplet (corresponding to its mean interaction energy) with the capacitance  $C_j$ . The second term describes the tunnelling of particles between the droplets with the Josephson amplitude  $J_j$ . The JJA model is appropriate if the droplets are reasonably well separated in space. For simplicity, we assume that  $C_j$  and  $J_j$  are identical for all the droplets and later denote them as C and J. Note that this Hamiltonian describes a quantum evolution since  $N_j$ and  $\theta_i$  are connected via canonical commutation relations.

We describe the time evolution of the system via a Langevin formalism<sup>35,36</sup>. The phase of the droplet *j* follows

$$\eta \frac{d\theta_j}{dt} = J[\sin(\theta_{j+1} - \theta_j) - \sin(\theta_j - \theta_{j-1})] + \xi_j(T, \eta, t), \qquad (2)$$

where the friction parameter  $\eta$  is a phenomenological way to account for dissipative mechanisms. The temperature *T* is introduced via the thermal noise  $\xi_j(t)$ , which shows Gaussian uncorrelated fluctuations at times *t* and *t'* given by  $\overline{\xi}_j(t)\xi_i(t') = 2\eta k_{\rm B}T\delta_{ij}\,\delta(t - t')$ , where  $\delta_{ij}$ is the Kronecker delta and  $\delta(.)$  denotes the Dirac distribution. The thermal energy scale  $k_{\rm B}T$  (for *T*=150 nK in the experiment and the Boltzmann constant  $k_{\rm B}$ ) is much higher than the estimated capacitance effect, allowing to neglect the second-order time derivative term related to *C* in the evolution of equation (2) (Methods).

This JJA model provides an intuitive understanding of the dephasing and rephasing dynamics shown in Figs. 2 and 3, respectively. It encapsulates the essential physics of a rigid droplet situation at a finite temperature *T* in terms of the two phenomenological parameters *J* and  $\eta$ . The timescale of de- and rephasing is dictated by the dissipation  $\eta$ . The dissipation mechanisms include atom losses (cf. Extended Data Fig. 3), energy and particle exchange with the thermal component of the gas or with some internal degrees of freedom of the droplets, as discussed later. In contrast, the phase fluctuations in the equilibrium state, namely, the stationary value of  $\Delta \phi$ , are set by the competition between *J* and *T* and are independent of  $\eta$ .

We develop a parameter-free theory–experiment comparison for the rephasing dynamics by first fixing the value of *J* and  $\eta$  from

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independent measurements. Taking advantage of the fact that  $\eta$  and J play different roles at different stages of the protocol, we fix J by reproducing the  $\Delta \Phi$  measured at the end of evaporation (Fig. 1d). The parameter  $\eta$  is instead extracted from the dephasing dynamics of  $\Delta \Phi$  during  $t_s$  (Fig. 2 and Methods). We find that J=6,000 Hz and  $\eta$ =60  $\pm$  10.

With all the values of the parameters fixed, we now compare the rephasing dynamics from the JJA theory and the experiment. As shown in Fig. 3, we observe that despite its simplicity, the model qualitatively predicts the rephasing curve well. In particular, the plateau value at large  $t_h$  as well as the time to reach the plateau are consistent with the experiment. In addition, we note that in the early time evolution ( $t_h < 30$  ms), the experimental data is systematically slightly above the JJA curve, namely, the system seems to rephase slower than predicted by the model. This indicates that other phenomena beyond our rigid JJA model are important to fully capture the out-of-equilibrium physics.

To elucidate why the observed experimental rephasing is slower than expected from the JJA model, we simulate the system's real-time evolution (RTE)<sup>24</sup> in the simplified zero-temperature and zero-atom-loss case, with a quench sequence mimicking the experiment. Different from the phenomenological JJA model, the ab initio RTE approach can additionally account for the 'soft' nature of the supersolid, namely, the crystal and phase phonons<sup>31-33</sup>. As shown in Fig. 4a, the RTE simulation shows that the collective modes are initially excited by the interaction quench. The positions and heights of the high-density peaks evolve in time, especially during the initial tens of milliseconds, resulting in time-dependent density links. At longer times, we observe a damping of the collective motion, which can be attributed to an initial redistribution of the population from a few modes to many higher-lying modes over time. This suggests a possible dissipation mechanism of the phase excitations for our experiment. Even though the finite temperature, atom loss and presence of a normal component affect the precise dynamics and damping in the experiment, the RTE calculation strongly indicates that the droplet dynamics play an important role at early times. This is compatible with the observed deviations from the rigid JJA model.

In the experiments, we do not have direct access to the in-trap density evolution. However, by repeating our experiment in a tighter trap, which gives more distinct side peaks in the TOF interference patterns, we observe the indications of collective mode excitations. In Fig. 4b, we plot the time evolution of  $\Delta \Phi$  following the rephasing protocol. Here, on top of a global decrease in  $\Delta \Phi$ , a low-amplitude oscillating behaviour is evident, as highlighted by plotting the residuals of an exponential fit to the data. We extract an oscillation frequency of 50(5) Hz (cf. Extended Data Fig. 4 and Methods).

By comparing the predictions of the JJA and RTE approach to our experimental data, we conclude that the phase dynamics is largely described by a dissipative and 'rigid' JJA picture. We speculate that the phase excitations primarily dissipate via coupling to excited modes<sup>37</sup>. Moreover, the droplet dynamics seems to play an important role, affecting the rephasing efficiency. Note that one could modify the JJA model to take the droplet dynamics into account, at least to a certain degree (Methods). Such coupling between the two types of degree of freedom usually leads to additional dissipation channels<sup>38</sup>. Besides being of intrinsic theoretical interest, this could, for example, open the perspective to using supersolids to study similar dynamics in regular solids between electronic and phononic degrees of freedom. This, however, goes beyond the scope of the present work and will be addressed in future studies.

In a final set of experiments, we investigate the role of the density-link strength between the droplets, namely, the Josephson coupling, in the rephasing dynamics. After phase scrambling (Fig. 1b, middle), we quench  $a_s$  to different values and record  $\Delta \Phi$  as a function of the hold time  $t_h$  (Extended Data Fig. 5a). For each  $a_s$ , we quantify the strength of the density link via  $\mathcal{L}$ , which was

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**Fig. 4 | Out-of-equilibrium dynamics. a**, RTE simulation starting from a zero-temperature cloud of  $2 \times 10^4$  atoms of <sup>164</sup>Dy and performing a scramble-and-rephase sequence. We plot the 1D integrated density profile (colour code) along the weak trap axis as a function of time. Starting from the SSP, the scattering length is linearly ramped at 1 ms to the 1D phase (first dashed line), held there for  $t_5$  = 20 ms (second dashed line), ramped back to the SSP in 1 ms (third dashed line) and then simply held for  $t_n$ . The trap frequencies are  $\omega_{xyz} = 2\pi \times (229, 37, 135) \text{ s}^{-1}$ . **b**, Experimental rephasing data for the same trap. The upper panel shows  $\Delta \Phi$  calculated analogously to that shown in Fig. 3 (black dots), as well as an exponential fit (grey line). The lower panel shows the residuals of the fit before (black line) and after (red line) the application of a numerical low-pass filter.

determined from a ground-state calculation (Fig. 1a). We study the short- and long-time evolution of  $\Delta \Phi$  by the determination of the initial rephasing rate |R| (the slope of a linear fit to  $\Delta \Phi$  for  $t_h \leq 20 \text{ ms}$ ) and the saturation value  $\langle \Delta \Phi \rangle_{\text{sat}}$  (simple mean of  $\Delta \Phi$  for  $t_h > 50 \text{ ms}$ ), respectively, as shown in Fig. 5a,b (for  $A_{\text{M}}$  and  $A_{\phi}$  see Extended Data Fig. 5b).

We observe different rephasing dynamics depending on  $\mathscr{L}$ . For extremely weak density links ( $\mathscr{L} < 10^{-3}$ ), which we associate with the ID regime, the system is unable to rephase and it remains incoherent over the whole time evolution as indicated by a low |R| and large  $\langle \Delta \Phi \rangle_{\rm sat} \approx 0.9$ . As  $\mathscr{L}$  slightly increases, the system shows a partial rephasing with  $\langle \Delta \Phi \rangle_{\rm sat}$  decreasing to about 0.5. By further increasing the density-link strength, there exists a critical value of  $\mathscr{L}$  above which the system recovers its full phase coherence after a long time ( $\langle \Delta \Phi \rangle_{\rm sat} = 0.15$ ). In this regime,  $|R| \approx 30 \, {\rm s}^{-1}$  is large and seems relatively independent of  $\mathscr{L}$ .

To compare the JJA predictions with our experimental observations, we first extract J for each  $a_s$ . For this purpose, we look at the long-time behaviour after scrambling and rephasing when the system has equilibrated, matching the theoretical and experimental  $\langle \Delta \Phi \rangle_{sat}$  values. This is justified by the long-time agreement observed in Fig. 3. As expected from the increasing density-link strength  $\mathscr{L}$  between the droplets, J globally grows with  $a_s$  (Fig. 5a, inset). Using these J values and the single calibrated value of  $\eta = 60$  (Fig. 2), we extract the rephasing rate  $|R|_{IJA}$  from the short-time evolution of  $\Delta \Phi$  within our JJA framework (Fig. 5b, inset). Despite the simplicity of the JJA approach, the  $|R|_{IJA}$  values are of the same qualitative behaviour with respect to  $a_s$ . As observed in Fig. 3,  $|R|_{IIA}$ 

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Fig. 5 | Density-link dependence of rephasing dynamics. a, Saturation value of the phase variance  $\left< \Delta \varPhi \right>_{
m sat}$  as a function of the scattering length  $a_{\rm c}$  and the corresponding  $\mathscr{L}$ . We calculate  $\langle \Delta \Phi \rangle_{\rm cat}$  and the error bars as the mean and standard deviation of  $\Delta \Phi$  over the three longest hold times (Fig. 3 and Extended Data Fig. 5). The grey-shaded rectangle indicates the theoretical  $1\sigma$  confidence interval for uniformly random phases. The inset shows the J values for which the JJA simulations yield a  $\langle \Delta \phi \rangle_{...}$ matching the experimental value at  $a_{s}$ , and the grey shading represents its confidence interval. **b**, Initial rephasing rates |R| for the different  $a_c$ values as extracted from the experimental data by using a linear fit. The error bars are the  $1\sigma$  confidence intervals of the fitted slopes. The inset shows the theoretical  $|R|_{IIA}$  (*a*<sub>c</sub>) expected from a linear fit to the initial decrease in the JJA rephasing curves (theory line in Fig. 3); the grey shading represents the confidence interval. The red filled points correspond to the dataset shown in Fig. 3.

generally predicts rephasing faster than that measured in the experiment, suggesting the presence of non-negligible processes beyond the rigid JJA model.

It is interesting that despite the JJA being able to consistently model the phase dynamics in the experiment, the extracted dependence of J on  $a_s$  is mild in comparison to the expected ground-state density link  $\mathscr{L}$  (Fig. 5a). For instance, J changes only by a factor of two whereas  $\mathscr{L}$  changes by two orders of magnitude. Further, J seems to effectively saturate for increasing strength of the ground-state density link  $\mathscr{L}$ , which is in agreement with the observed plateaus of both  $\langle \Delta \Phi \rangle_{sat}$  and |R| for large  $a_s$ . Possible explanations include the breakdown of the JJA model for a low-contrast supersolid, the impact of finite temperature on both the equilibrium supersolid state itself and the experimental measurement, or the role of collective dynamics.

In conclusion, we have performed a study of the out-of-equilibrium dynamics of a dipolar supersolid after an interaction-driven phase excitation that fully destroys its phase coherence. We have demonstrated that if the inter-droplet density links are sufficient, this phase-scrambled system relaxes into an equilibrium phase-coherent state. With decreasing link strength, the rephasing substantially slows down and eventually ceases in the ID regime. We find an overall consistency between the phase dynamics observed in the experiment and an intuitive, theoretically easily tractable rigid JJA model. However, both ab initio simulations and experimental observations indicate post-quench collective excitations of the droplet array, which can affect the phase dynamics. Our study shows the evidence of particle flow across a dipolar supersolid, connecting to its superfluidity. It also suggests the efficient dissipation of phase excitations,

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whose microscopic mechanism is still under question. Future experimental works, combined with advanced out-of-equilibrium theoretical models, will be crucial to understand the relaxation dynamics at play in isolated and open supersolid states of quantum matter.

#### **Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/ s41567-020-01100-3.

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**Extended Data Fig. 5** | Dependence of experimental rephasing dynamics on the density link strength  $\mathcal{L}$ . a, Temporal evolution of  $\Delta \Phi$  (color map) at different  $\mathcal{L}$  starting from phases scrambled in the ID regime. For each  $t_h$  we record  $q \ge 35$  individual experimental realizations. For large  $\mathcal{L}$  the system recovers its global phase coherence ( $\Delta \Phi \simeq 0$ ), whereas for small  $\mathcal{L}$  it does not ( $\Delta \Phi \simeq 1$ ). b,  $A_M$  (circles) and  $A_{\phi}$  (diamonds) for the same data set at long hold time,  $t_h = 100$  ms. The error bars (partly covered by plot markers) are statistical standard errors of  $A_M$  and  $A_{\phi}$ . The red filled pair of symbols corresponds to the data set presented in Fig. 3.

Part III

# A DIPOLAR QUANTUM GAS MICROSCOPE

For any man with half an eye, What stands before him may espy; But optics sharp it needs I ween, To see what is not to be seen.

– John Trumbull, "McFingal", Canto 1 (1775–1782)

#### MOTIVES FOR MICROSCOPY

Interference between intersecting laser beams can produce standing waves of light, and hence, via the optical dipole force, a periodic trapping potential for atoms [118], molecules [39] or even macroscopic composite particles [10, 11, 78]. Optical lattices serve as an important tool in quantum optics, either for studying many-body physics in periodic potentials and potential applications as quantum simulators [34, 35], or simply as a means of keeping particles in large ensembles separate from each other, which is important, e.g., to reduce interaction effects in optical clocks [186].

For a long time, to access the physics of atoms in optical lattices, experimentalists had to rely on time-of-flight methods, where the particle correlations and lattice periodicity are only mirrored in momentum space [101, 114]. This changed with technological advancement and the experimental realisation of quantum gas microscopes, which allow to image single atoms on individual sites of a single 2D lattice plane. Working quantum gas microscopes have been first demonstrated in 2009/10 in two poineering experiments at Harvard and Munich, both using the bosonic <sup>87</sup>Rb isotope [18, 267].

With this new technology, it has become possible to study many-body physics on the single-particle level, including quantum phase transitions [18, 267], correlations [58], particle dynamics [105, 238] and other, similarly fundamental effects (see, e. g., Refs [121, 160] and references therein). About five years later, two Japanese groups demonstrated single-site resolution with the bosonic lanthanoid species <sup>174</sup>Yb [197, 297], whose complex level structure held promise for the realisation of new quantum information protocols.

In 2015, microscope experiments for fermionic atoms of five different research groups went into operation, using either <sup>6</sup>Li [211, 214] or <sup>40</sup>K [60, 84, 129]. Their development had been more demanding than for the alkali bosons because of greater difficulties in optical cooling due to the small hyperfine splitting and the low atomic mass [121, 211]. Since then, as expected, also the fermion microscopes have produced results of great impact, such as the direct observation of band [211] and (fermionic) Mott insulators [60, 113], antiferromagnetic ordering [40, 215], and many more (see, e. g., the recent review of Gross & Bakr [122]).

All quantum gas microscopes realised so far use atomic species with a negligible magnetic dipole moment. The atoms therefore interact only via the short-range contact interaction, such that they do not feel a direct energy shift depending on whether a neighbouring site is occupied or not. Interaction effects then only manifest in an occupancy-dependent modification of tunneling rates.

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Molecules on lattices Direct long-range interactions between particles on neighbouring sites would present an enriching addition to this setting. This has been the motivation for several groups trying to produce ground-state hetero-atomic molecules in an optical lattice, which would posses a large *electric* dipole moment and therefore feature strong, long-range dipole-dipole interactions. However, technical difficulties related to the cooling, association, and reactive losses of the molecules have made the realisation much more difficult than initially expected [39]. Despite very recent important improvements [274, 286], this still presents a severe limitation and the molecule experiments are still far below the level of control achieved in experiments with singular atoms.

Magnetic atoms on lattices

Possible research directions

A promising experimental approach therefore consists in the use of strongly *magnetic* atoms such as erbium and/or dysprosium, which combine the simpler cooling techniques and low losses of atomic systems with long-range dipolar interactions. Some effects of long-range dipolar interactions on lattice physics have, for example, already been studied in our group using fermionic <sup>167</sup>Er and well-established TOF techniques [14, 216]. Ultimatively, it would be very desirable to perform experiments with dipolar atoms on a lattice in a quantum gas microscope, where many phenomena are much more directly accessible.

There exists a wealth of promising research proposals that could be approached using such a system. Interesting examples include, first, the possibility to observe lattice neighbour interactions via the long-range DDI deep in the Mott insulator regime, where particle tunneling (and hence contact interaction) is completely suppressed. A second possible direction is to study the formation of magnetic domains when dipolar atoms on a lattice are cooled down in absence of an external magnetic field. A third proposed experiment is more specific to the Innsbruck erbium-dysprosium mixture apparatus. In such a doubly-dipolar mixture one could employ a lattice that is at magic wavelength for the first species, such that the lattice potential will only be seen by the second species. The atoms of the second species can then be pinned in the optical lattice sites and interact with the first species via DDI. The first species therefore sees an effective periodic potential that is created by the pinned, second species. Since the pinned atoms can vibrate, DDI-mediated periodic potential itself supports phononic excitations, in contrast to a conventional optical lattice, which is infinitely stiff. Therefore, such a DDI-mediated lattice would presents a novel and much more realistic quantum simulator for, e.g., electrons in a metal.

Evidently, these are only three obvious, promising research directions. Many more already exist or might become apparent once a dipolar quantum gas microscope is in operation.

In the following chapters, I will describe the design of a new type of quantum gas microscope that I have designed and engineered as an addition to our group's erbiumdysprosium experiment. It is housed in a new section of the vacuum apparatus that will be attached to the existing machine (cf. § 5.1) via a mechanical UHV gate valve.

#### MICROSCOPE OPTICS

In this chapter, some important requirements and constraints that were decisive in the design process of the Er–Dy quantum gas microscope are sketched.

#### 11.1 REQUIREMENTS AND OPTIONS

Quantum gas microscopes are highly advanced and complex machines, often operating at the edge of what is technologically possible. Therefore, the optical design typically needs to be carefully tailored to the experimental needs.

In the case of the Innsbruck erbium-dysprosium experiment, we identified the following key requirements for the imaging system:

- 1. A high numerical aperture (NA) > 0.8 to resolve lattices of small spacing ( $\leq 532$  nm);
- 2. a large working distance on the order of millimetres to allow optical access, to grant full freedom of lattice wavelengths, and to avoid possible effects of close surface on the atoms;
- 3. use of exclusively non-magnetic and non-conducting materials to avoid magnetisation and eddy currents;
- 4. achromaticity at 401 and 421 nm (at least).

Let us take a brief look at what design options were initially considered for the Er–Dy microscope.

When a sample in a UHV environment is imaged, the vacuum window necessarily becomes part of the light path and needs to be considered in the optical layout. The aberrations introduced by a plane-parallel glass plate are mainly spherical and chromatic, scaling with the thickness of the plate [193, 269]. Different quantum gas experiments have found different solutions for dealing with these aberrations, which can be grouped in four categories as sketched in Fig. 14a–d and summarised in Table 2 on page 107.

The most straight-forward option is design (a), where the objective is simply mounted outside the vacuum chamber, as for our main-chamber imaging (§ 5.2). Such a design in general features a long working distance on the order of some to tens of millimetres which, typically, also limits the attainable NA to below  $\sim 0.7$ . For a sufficiently thin glass plate ( $\leq 1$  mm), the aberrations introduced by the glass window can be still low enough to use a commercial objective, as in the Toronto microscope [84]. However, the bending of a thin plate under the pressure difference between in- and outside limits this approach to very small window diameters. Hence, the more frequent approach is to use a custom objective carefully corrected for the window, such as in the microscopes in Munich [211, 267] or Kyoto [297].

A second design option (Fig. 14b) uses lenses in shape of a truncated sphere in very close distance to the sample. Such a lens leads to an enhancement of the NA by a factor

Imaging concepts for UHV

Key requirements



FIGURE 14: *Types of microscope objectives applied in quantum gas experiments.* (a) Objective outside vacuum chamber, (b) solid-immersion lens (SIL) close to sample, (c) part of objective optics in vacuum, (d) entire objective in vacuum. Cf. Ref. [193].

that depends on the truncation; this is called the solid-immersion effect [190].<sup>1</sup> Hemispherical solid-immersion lenses (SILs), for which the enhancement factor is equal to its index of refraction,  $n_r$ , have been used in configurations with an in-vacuum lens mount [18] or with the lens optically contacted to the glass window [60, 197, 214]. The latter design bears the advantage that the window becomes part of the SIL and as such does not produce aberrations anymore. Another type of SIL, demonstrated in Bonn [247], uses a Weierstraß sphere and offers an even higher NA enhancement factor of  $n_r^2$ . A modified version of such a microscope is currently under construction in Innsbruck.<sup>2</sup> The drawback of solid-immersion lenses is their small working distance, requiring a very close placement of the sample (typically micrometres) in front of the lens surface. This necessitates complex transport strategies to bring the atom close to the focus plane, and usually requires that both, horizontal and vertical lattice beams are reflected off the front surface, with corresponding high requirements for the optical coating.

A third design option (Fig. 14c) uses a first optical element inside the vacuum chamber which accepts the light at high NA, reducing the angles of incidence on the vacuum window and hence the aberrations caused. Outside the vacuum chamber, the light cone can then be collected by an objective with lower NA. Imaging systems of this type have been demonstrated in Palaiseau [275] and Innsbruck [193], with moderate NAs around 0.5. The disadvantage of this design is that it depends crucially on the relative alignment of the in- and ex-vacuum components. This poses a great challenge to alignment strategies in general, as well as for the mounting concept to exclude any relative drifts or vibrations.

A fourth design option (Fig. 14d) that does not suffer from this problem is an objective entirely in vacuum. While being conceptually simple, it allows for both a millimetre-level working distance and high NA. If the objective is at infinite conjugation, i. e. the rays exit in parallel, the glass window does not add any aberrations. Despite these advantages, this design is less frequently encountered in cold-atoms experiments due to the technical difficulties of (i) vacuum compatibility of the objective itself and (ii) the requirement of a large-volume vacuum chamber to contain it. Nevertheless, successful objectives of this type have been realised in Orsay [257] and recently in Munich [183].

<sup>1</sup> Irrespective of the enhancement factor, the NA is of course still bounded to below unity. This can only be exceeded using so-called 4Pi microscopes [131], which are applied in the cold-atom context, for example, in Hamburg [140, 304].

<sup>2 &#</sup>x27;NewLanD' dysprosium experiment by Emil Kirilov & Rudolf Grimm.

|      | 8 71                                 |                                                     |
|------|--------------------------------------|-----------------------------------------------------|
| ΤΥΡΕ | ADVANTAGE(S)                         | DISADVANTAGE(S)                                     |
| (a)  | easy alignment<br>long WD            | limited NA                                          |
| (b)  | high NA<br>mechanically rigid        | short WD<br>lattice wavelength fixed by coating     |
| (c)  | intermediate NA<br>intermediate WD   | difficult alignment<br>possible relative vibrations |
| (d)  | high NA<br>easy alignment<br>long WD | vacuum considerations                               |
|      |                                      |                                                     |

TABLE 2: Design types of quantum gas microscopes.

Objectives completely outside vacuum (Fig. 14a) can hardly deliver the NA and resolution aimed for. It has further been a determining requirement that our objective has to work with both species, erbium and dysprosium. This poses strict demands for the layout of the imaging optics themselves, but also, in particular, on optical coatings of other elements involved. For example, we will need various lattice beams (including standard and magic-wavelength lattices, possibly optical superlattices), other beams for lattice loading and a specific shaping of the optical potential (possibly including DMDs), probably cooling beams and beams for spin manipulation. All these laser beams need to be directed onto the sample. This requirement seemed incompatible with a SIL design (Fig. 14b), since a coating that is highly antireflective for the imaging wavelengths over a large angle range and at the same time highly reflective for many others, such as the lattice beams, is hardly feasible, especially since we wanted to keep the possibility to switch to even smaller (UV) lattice constants later; what will be the most promising UV wavelength is still a point under discussion. Such great flexibility concerning wavelengths is only possible if the lattice beams can be sent in from the side and do not have to be reflected off the lens surface, i.e. with a sufficient working distance (WD). The lens-in-vacuum design (Fig. 14c) seemed risky since, at the level of optical resolution aimed for, tolerances on alignment imperfections are minimal, and drifts in relative alignment or relative mechanical vibrations can be fatal for the performance.

For these reasons, despite the vacuum challenges and the technology developments it required, we opted for the objective-in-vacuum design (Fig. 14d).

#### 11.2 DESIGN OF THE ER-DY MICROSCOPE OBJECTIVE

Together with a company<sup>3</sup> specialised in manufacture of custom optics we have developped a high-NA objective for the imaging of erbium and dysprosium atoms (Fig. 15 on the following page). It is designed for achromatic performance on the broad, blue imaging transitions at 401 and 421 nm wavelength, as well as for 633 nm, which is the alignment wavelength of the manufacturing company and, by coincidence, close to the red dysprosium transition at 626 nm. The optical design has been carried out by ray-tracing and field Choice of strategy

<sup>3</sup> Special Optics, Inc., NJ/USA.

propagation (cf. § B) techniques using the commercial software Zemax<sup>®</sup>. The objective design contains five singlet lenses and no cemented elements, to avoid vacuum problems caused by outgassing. The five lenses are custom-manufactured from different glasses that have a sufficient transmission for our blue wavelengths. A mechanical drawing is presented in Fig. 15.



FIGURE 15: Cross-sectional drawing of the in-vacuum objective for the Er–Dy experiment. Dimensions are in millimetres. Figure data by courtesy of Special Optics, Inc.

The predicted values of the optical characteristics from our simulations are summarised in Table 3; moreover, some important simulated quantities are plotted in Figs 16–17.

| CHARACTERISTIC                                | UNIT | PREDICTED VALUE |        |  |  |
|-----------------------------------------------|------|-----------------|--------|--|--|
| effective focal length                        | mm   | 20.0            |        |  |  |
| total length                                  | mm   | 70.0            |        |  |  |
| chromatic focal shift                         | μm   | 0.39            |        |  |  |
| working f-number <sup>a</sup>                 |      | 0.56            |        |  |  |
| object-space NA                               |      | 0.89            |        |  |  |
|                                               |      | 401 nm          | 421 nm |  |  |
| object-space Airy disc radius                 | μm   | 0.274           | 0.288  |  |  |
| wavefront error <sup>a</sup> peak to valley   | λ    | 0.098           | 0.107  |  |  |
| wavefront error <sup>a</sup> root mean square | λ    | 0.0321          | 0.021  |  |  |
| Strehl ratio <sup>b</sup>                     |      | 0.96            | 0.98   |  |  |
| Ø diffraction-limited FOV                     | μm   | 160             | 180    |  |  |

TABLE 3: Important characteristics from the simulation of the imaging objective.

<sup>a</sup> Evaluated over full pupil.

<sup>b</sup> On axis (zero field).

Miniature mirror

The objective features a miniature mirror<sup>4</sup> of 1.5 mm diameter, glued to the front lens using a high-performance UHV-compatible adhesive<sup>5</sup>, see Fig. 18a on page 111. The miniature

<sup>4</sup> Manufactured by Optics Technology, Inc., NY/USA.

<sup>5</sup> Optocast 3415 from Electronic Materials, Inc., CO/USA.



FIGURE 16: *Calculated characteristics of the imaging objective for the erbium and dysprosium imaging wavelengths*. Dashed lines represent the diffraction limit according to the Maréchal criterion [191].

mirror reduces the maximum NA of 0.89 by only ~ 3%, so hardly affects the resolution limit, and will serve to reflect off the vertical lattice beams such that the lattice position relative to the objective is fixed. This is important to avoid relative drifts between the lattice plane position and the focal plane, which could lead to severe defocus effects with regard to the small depth of focus  $\Delta z$ . The depth of focus can be defined as the on-axis distance that introduces a  $\lambda/4$  wavefront error [180] and accounts to

$$\Delta z = \pm \frac{\lambda}{2} \left(\frac{1}{NA}\right)^2 \approx \pm 0.25 \,\mu \text{m} \tag{97}$$

given our imaging wavelengths and maximum NA.

The objective lens tube and all lens retaining mechanics are fully fabricated from machinable ceramics (Macor<sup>®</sup>), see Fig. 18a on page 111. The objective is therefore non-magnetic and non-conducting by design, and we avoid magnetisation effects and eddy currents, which could deteriorate the magnetic environment close to our magnetic atoms as well as limit the field-switching times.

All volumes inside the housing and in between lenses have to be vented through borings to avoid virtual leaks under UHV.

The objective tube will be mounted in a home-built Macor mount, shown in Fig. 18b on page 111. It comprises three titanium flat springs around its perimeter and sits on three ruby balls to define the objective position within the glass cell (cf. § 12.2).

Around the top rim of the mount, in the future lattice plane, an arrangement of solid quartz UV-enhanced aluminium mirrors has been engineered in custom geometry.<sup>6</sup> We colloquially refer to them as the 'crown mirrors'. Eight crown mirrors (round in Fig. 18b)

Objective mount

<sup>6</sup> Manufactured by Optico AG, Switzerland.



(a) Tangential fan of OPD compared to chief ray vs normalised pupil coordinate  $P_y$ . Solid (dashed) lines correspond to zero (40 µm) lateral field along *y*. Sagittal fans are independent of *y*-field and equal to the tangential fan at zero field.

(b) Modulus of the OTF. The solid black line presents the diffraction limit. Magenta (cyan) corresponds to to zero (40  $\mu$ m) lateral field along *y*. Solid (dashed) lines correspond to tangential (sagittal) direction. *Insets:* A zoom-in for intermediate spatial frequencies.

FIGURE 17: Optical path difference (OPD) and modulus of the optical transfer function (OTF) for the microscope objective at 401 and 421 nm.

are in staggered alignment with the side windows of the glass cell. They will serve to reflect laser light onto the atoms, entering and exiting through the large top window. One crown mirror (rectangular in Fig. 18b) will protect the objective itself from being hit by the divergent transport beam when the focus is at the main-chamber centre. This is important to avoid dumping optical power on the objective housing, which could lead to drifts in temperature and optical performance.

The individual pieces of the mount are assembled using vented titanium screws and beryllium-copper disc springs which can take up mechanical stress upon temperature changes, such as during bake-out.



(a) The microscope objective with the small lattice mirror (*inset*).



(b) The assembled objective mount with the crown mirrors.

FIGURE 18: In-vacuum optics for the quantum gas microscope.

#### 11.3 MICROSCOPE PERFORMANCE TESTS

The optical performance of the microscope objective was tested by imaging the tip of a SNOM fibre, as described in § 5.2.5 and Refs [155, 247], using a stock 1000-mm achromat<sup>7</sup> as imaging lens at infinite conjugation. The images close to focus, the azimuthally averaged spot profiles and fitted Gaussian functions are shown in Fig. 19. These measurements yielded a resolution limit  $d_0$  according to the Rayleigh criterion (Airy disc radius) of

$$d_0 = \begin{cases} 0.29(1)\,\mu\text{m} & \text{for} \quad \lambda = 401\,\text{nm}, \\ 0.30(1)\,\mu\text{m} & \text{for} \quad \lambda = 421\,\text{nm}, \end{cases}$$

close to the values predicted from our simulation (Table 3 on page 108).

To find the focus position, it is often convenient to consider power-normalised images. From the point of energy conservation, aberrations do not change the amount of total power that arrives at the image plane, but manifest in a departure from the Airy pattern of a diffraction-limited system through a redistribution of power from the central peak to regions further out. Hence, as long the magnification of an imaging system does not change, its Strehl ratio is proportional to the intensity maximum of the power-normalised image [36]. The peak value in power-normalised images can therefore be used as a figure of merit during the alignment of an optical system, which has been used extensively in the course of this thesis.

Figure 20b shows images from a focus scan, where the SNOM fibre tip has been moved along the optical axis using a piezo actuator<sup>8</sup>. In the corresponding normalised intensity (Fig. 20a), we observe maxima corresponding to the focus positions for 401 and 421 nm wavelength. The distance between these maxima, the chromatic focal shift, is  $\sim 2.4 \,\mu\text{m}$ .

1

images

Power-normalised

Chromatic focal shift

Chromatic focal shift

<sup>7</sup> ACT508-1000-A, Thorlabs, Inc., NJ/USA

<sup>8</sup> PiezoMike N-472 from Physik Instrumente (PI) GmbH, Germany



FIGURE 19: Resolution test of the microscope objective. Images of a SNOM fibre tip (respective left) for distances (x, y) w.r.t. the object plane. The corresponding azimuthally averaged intensity profile (respective right) is plotted vs the radial coordinate  $(\rho)$ . The red lines are Gaussian fits.

The objective is therefore close to, but not fully achromatic, as initially demanded. According to the manufacturer, this is most probably due to insufficient accuracy of their prior knowledge of refractive indices of the lens glasses, which had to be extrapolated down to 401 nm wavelength, where the dispersion is very steep. For imaging of only one species per time, this does not pose a problem, since the shift can be corrected by re-adjusting the camera position. For a simultaneous double-species imaging, the chromatic shift needs to be circumvented. Possible experimental solutions include (i) to use two separate beam paths and cameras for the two species after the objective, using a beam splitter, or to image both species shortly after each other in combination with either (ii) an adaptive optical element, such as a focus-tunable lens, (iii) an imaging lens on a fast translation stage to dynamically adjust the focus position, or by (iv) dynamically shifting the vertical lattice position with the atoms between the respective focal planes.





(a) Normalised intensities of the spot in dependence of on-axis position (z). Values for 401 nm in blue, 421 nm in red. Solid lines are Gaussian fits to the maxima; the dashed lines give the centres, the grey shadings the standard deviations.

(b) Through-focus series for 401 nm (*top row*) and 421 nm (*bot-tom row*). The images in the central column are the ones closest to the respective focus and correspond to Figs 19a, b.

FIGURE 20: *Measurement of the chromatic focal shift*. For wavelengths of 401 and 421 nm, the measured distance between the foci is around  $2.4 \,\mu$ m.

The light from the atoms is collected by the microscope objective and passes the vacuum window as a collimated beam. To form an image, this beam must be re-focussed on the camera chip. For the image sampling to accord with the Nyquist-Shannon criterion (Eq. 95), a sufficient image magnification M is needed, resulting in a minimum effective focal length f' for the focussing system. For example, if we want to achieve a sampling of five pixels per lattice site with a sensor pixel size of  $d_{px} = 16.5 \,\mu\text{m}$ , a lattice constant of 0.266  $\mu\text{m}$  and using our objective focal length of  $f = 20 \,\text{mm}$ , it follows  $M \ge 310$  and  $f' \ge 6.2 \,\text{m}$ . For larger lattice spacings or smaller pixel size the numbers are smaller, but still likely on the level of several metres. Such long light paths would naturally suffer from stability problems caused by mechanical vibrations or air currents.

For this purpose, a compact telefocus system in Kepler configuration has been developped which offers a large effective focal length at a small physical size. Our imaging simulations show that with an arrangement consisting solely of stock lenses<sup>9</sup> an effective focal length of 6.2 m can be achieved in a system that itself has only a length of around 1.1 m while being fully achromatic at 401 and 421 nm wavelength.

#### A Kepler telefocus system

<sup>9</sup> One long-focal-length achromatic doublet (many available from Thorlabs, Inc., to match the required magnification) and one short-focal-length Hastings achromatic triplet.

As already touched upon in § 5.1, experiments with ultracold atoms have to be conducted under ultra-high vacuum (UHV) on the order of  $10^{-11}$  millibar to isolate the samples from the environment and permit sufficiently long lifetimes. Such low pressures can only be attained and maintained using specially suited materials, techniques, and instrumentation [144].

#### 12.1 GENERAL VACUUM CONSIDERATIONS

#### Vacuum Materials

The demand for UHV puts strict requirements for the building materials.<sup>1</sup> They must possess a low permeability for atmospheric gases, a low vapour pressure and outgassing of contaminants (such as H<sub>2</sub>, CO and CO<sub>2</sub>), and a high-quality, low-roughness surface finish. This restricts the choice of materials to certain metals, ceramics, glasses, and some high-performance organic polymers such as Kapton [154], which is sometimes used for electrical insulation inside UHV.

Since in our experiment the magnetic behaviour is of high importance, the main building material for the vacuum apparatus are the stainless steels 316LN and 316L.<sup>2</sup> They offer (i) a low relative magnetic permeability, hence do not easily get magnetised when magnetic fields are applied to the atomic sample, and additionally (ii) a high electric resistance, which reduces eddy currents upon changes of the magnetic through-flux and enables faster field switching times. For a reduction of surface area and the concurrent outgassing rates, it is advantageous to apply mechanical or electropolishing to the vacuum faces of metallic parts, and to glow them at over 1000 °C under vacuum for some hours to reduce the content of  $H_2$  [119, 144].

Where optical transmission is required, most glasses are good vacuum materials [71]. When high powers or short wavelengths are needed, the preferred material is fused silica, a synthetic quartz glass. It features low light absorption down to below 400 nm, and little thermal lensing effects even at high light intensity.

Machined parts that have to be non-metallic, e. g. for electrical insulation, lower thermal expansion, or complete suppression of eddy currents and magnetisation, are preferentially manufactured from Macor, a glass-ceramic that can be machined using standard metalworking tools [71]. Using special techniques and tools, normally only available in specialised companies and workshops, also quartz can be machined directly. In the microscope setup, Macor is used for the microscope housing and mount, whereas machined and polished quartz is used for the crown mirrors and the octagon corpus of the glass cell.

Before the final implementation, all machined components have to undergo a thorough cleaning procedure [144]. After the initial removal of dust and watery contaminants, de-

Glass

Ceramics

<sup>1</sup> A useful and extensive list of materials suitable for use in UHV environments has been published by the LIGO collaboration [71].

<sup>2</sup> American Iron and Steel Institute (AISI) classification.

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greasing steps using detergents and organic solvents such as acetone, ideally in an ultrasonic bath for several minutes, are of vital importance to eliminate oils and other pollutants from the manufacturing process.

#### Vacuum seals

Metal-to-metal seals Forming a UHV seal between two components requires appropriate techniques. For joining steelparts, the laboratory standard is to use all-metal seals, formed by knife-edge flanges which cut into a soft gasket manufactured from oxygen-free copper. Where UHV connenctions need to be opened and closed repeatedly, also reusable all-metal valves relying on a tight, mechanical metal-to-metal contact are commercially available.<sup>3</sup>

Glass-to-metal seals Glass-to-metal seals, for example for vacuum windows, are nowadays mostly formed by specialised companies. They typically use a braze alloy which, in molten form, wets the prepared viewport glass and forms a tight seal when cooled down and solid. A crucial point is that the thermal expansion coefficients of metal fitting and viewport glass may not differ too strongly, otherwise temperature changes (which are not completeley avoidable, e. g. in the production process or during vacuum bake-out) could lead to stress and – in the worst case – cracking of the glass. Commercial standard fittings therefore often employ a flexible-lip design and soft braze materials that can take up part of the stress. Still, not all metal-glass combinations are feasible. For example, stainless steel and quartz glass are largely incompatible, unless gradient-index transitions are formed, where by use of different glasses the coefficients of expansion are gradually matched between the two end materials. However, such gradient-index transitions are typically long (around 10 to 20 cm) and mechanical weak points, so their application is not permissible without restrictions.

A quartz-to-steel seal that can be formed in the lab without special equipment is based on indium metal as a sealant [120, 146, 291]. Indium is a soft metal with a low melting point, low permeability, and low outgassing rates [71]. Exposed to air, it is covered by a thin, passivating oxide layer. When it is mechanically deformed, however, like when pressed onto a glass surface, this oxide layer breaks and fresh, reactive metal is exposed. This activated indium wets the glass and and reacts with it, forming a tight seal.

Glass-to-glass seals For permanent glass-to-glass seals, as have to be formed when assembling a multi-piece scientific glass cell, techniques that do not require adhesive glues are preferential for their better vacuum quality. Established techniques include heat-diffusion bonding, glass-frit bonding, optical contacting by pure polishing, and direct melting. Where it is necessary to form the seal at room temperature and/or reversibly, also an indium seal between two glass pieces can be formed, as will be necessary for closing our glass cell once the objective is inserted.

#### Vacuum pumping and quantification

As touched upon in §5.1 and laid out in detail in Ref. [141], in the Er–Dy experiment we use several complimentary techniques to create and maintain UHV. Usually, the initial evacuation step from ambient pressure to low vacuum is achieved using mechanical pumps. In the experiment, this is accomplished using detachable, external roughing and

<sup>3</sup> E. g., from VAT Vakuumventile AG, Switzerland; Pfeiffer Vacuum GmbH; Just Vacuum GmbH, Germany; Vacom GmbH, Germany; Kurt J. Lesker Company, PA/USA.

turbomolecular pumps. All later steps to higher vacuum rely on some sort of gettering process, where gas particles get caught by chemi- or physisorption. This category involves evaporable getters as in a titanium sublimation pump, non-evaporable getter (NEG) materials, and ion pumps. An ion pump is essentially a Penning trap containing moving electrons which can hit and ionise entering gas particles, which are in turn accelerated in the electric field and finally strike an electrode surface where they either get buried, chemically adsorbed, or sputter electrode material on the walls around, which then itself acts as a getter material [262].

Finally, to quantify the vacuum in an experiment, one typically relies on ionisation gauges such as the hot-cathode Bayard–Alpert gauge, which measure an ion current that depends on the gas pressure [144]. A related, indirect indication of the pressure in the apparatus is given by the current drawn by the ion pumps.

#### 12.2 THE MICROSCOPE VACUUM SECTION DESIGN

The basic geometry of the microscope vacuum setup is given by a stainless steel cross and a scientific glass cell (see Fig. 21).



FIGURE 21: Vacuum connection from main chamber to microscope cell. The optical transport axis for the atoms is indicated in green.

The horizontal arms of the custom steel cross connect the glass cell to the experimental main chamber over a gate valve and form part of the future transport distance for the atomic samples. The lower vertical arm connects to a NEG element<sup>4</sup>, whereas the upper arm connects to (i) a combined NEG/ion pump module<sup>5</sup>, (ii) an ionisation gauge<sup>6</sup>, and (iii)

Steel cross

<sup>4</sup> Model Capacitorr Z200, SAES Getters S.p.A., Italy

<sup>5</sup> Model Nextorr D200, Mu-metal-shielded, SAES Getters S.p.A., Italy

<sup>6</sup> Model UHV-24P, tungsten-filament Bayard-Alpert ionisation gauge, Agilent Technologies, Inc., CA/USA

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an all-metal angle valve<sup>7</sup> for attachment of external mechanical pumps (see Fig. 23). All metal vacuum pieces have been electropolished and vacuum-glowed<sup>8</sup> for reduction of H<sub>2</sub> outgassing as well as for reduction of magnetic permeability.

Glass cell

The glass cell<sup>9</sup> consists of an octagonal quartz corpus with large borings ( $\emptyset \sim 70 \text{ mm}$ ) at top and bottom and eight small borings ( $\emptyset \sim 22 \text{ mm}$ ) through the side facets. Attached to the bottom boring and one of the side borings are a small- and a large-diameter quartz tube with polished flat lips on their ends, see Fig. 22. Via glass-frit bonding, one 3" and seven 1" fused-silica windows are attached to the top and the remaining side borings, respectively. For maximum flexibility concerning the wavelengths to pass through, the windows are coated on the inside with a gradient-index antireflexion nanostructure coating<sup>10</sup>, and uncoated on the outside, where cleaning from dust, etc., might become necessary at some point.

FIGURE 22: The glass cell to house the microscope. Custom-manufactured from quartz (body) and fused silica (windows). Polished faces on left and bottom tube are for indium sealing. The viewports feature a broadband, broad-angle gradient-index nanostructure coating on the inside. For scale, the hole distance in the breadboard below is 25 mm.



The small-diameter quartz tube supports the glass cell and connects it to the steel cross via a glass-to-steel indium seal. For this, a novel type of disc-spring-loaded clamping mechanism and a flat-to-knive-edge adapter has been engineered. It allows to separate the critical step, the formation of the indium seal, from the following attachment of combined adapter and cell to the steel cross through a simple CF flange.<sup>11</sup>

The large-diameter quartz tube is used to insert the microscope objective and its mount (Fig. 24) into the glass cell. The horizontal position of the mount in the centre of the quartz tube is defined by three titanium flat springs (Fig. 24b). The tube is then closed by a fused-silica window, on which the mount rests via three ruby ball lenses<sup>12</sup>. An indium glass-to-glass seal between tube and window is formed and supported using another pair of disc-spring-loaded clamps.

Indium sealing

In the course of this thesis, considerable time and efforts have been afforded for the engineering of parts connected via indium seals, as well as the development and improvement of cleaning and handling procedures for the formation of the seals. Points that turned out to be very critical for indium UHV seals include:

12 Edmund Optics, Inc., NJ/USA

<sup>7</sup> Model 57.1, VAT Vakuumventile AG, Switzerland

<sup>8</sup> Carried out by Reuter Technologie GmbH, Germany.

<sup>9</sup> Manufactured by Precision Glassblowing of Colorado, Inc., CO/USA.

<sup>10</sup> RAR.L2 from Tel Aztec LLC, MA/USA

<sup>11</sup> Concerning these developments I am particularly indebted to Emil Kirilov, Innsbruck, for valuable discussions.


- Use of round indium wire<sup>13</sup>, activated in hydrochloric acid (37%) for around 1 min straight before use, and connection of the freshly cut, angled ends to an O-ring.
- Surface finish: metal contact surfaces should be lathed (not milled) with the stroke direction aligned with the indium wire; glass surfaces must not be matt, but have to be polished till optically clear and should remain uncoated.
- Cleaning: metal contact surfaces should undergo usual vacuum cleaning (water and detergent, then organic solvants); glass surfaces should be (i) cleaned in water and detergent, (ii) blow dryed, (iii) rinsed with acetone, (iv) rinsed with methanol, and (v) air-dryed.
- Seal formation: gently and evenly squeeze the indium O-ring between the two surfaces, ideally using a feeler gauge. If content, perform a helium leak test; small leaks may be closed by stronger squeezing, warming up of the seal region, or simply leaving the indium flowing for some hours [161].

If an indium seal needs to be re-opened, for example because it contains a large leak, this is best done by heating the whole seal region to above the indium melting point at 157 °C. Glass parts can be cleaned from molten indium by wiping with a tissue, and from residuals by soaking in hydrochloric acid.

Once the indium seals are formed and steel cross, cell, and all other parts of the microscope vacuum section are assembled, the assembly can undergo a gentle pre-bake-out outside our laboratory to reduce down-time for the Er–Dy experiment. This pre-bake-out

Vacuum assembly

<sup>13</sup> Ø 0.05"  $\approx 1.3\,\text{mm}$ , In 99.995%, Indium Corporation of America Co., MD/USA

while pulling vacuum serves primarily to remove water vapour and other volatile contaminants from the inner volume. The maximum temperature is limited to around 90 °C due to the in-vacuum objective, wherefore the pre-bake has to last for several weeks. For the glass cell and its interior parts, this process can be assisted by prior or accompanying UV/ozone cleaning [55, 130]. After the pre-bake-out, the whole section can be flooded with noble gas, moved to the experimental table, attached to the main experiment, and evacuated. Depending on whether the attainable vacuum level is already sufficient or not, another short, low-temperature bake-out on-site may be needed.



coils and shielding.

Optical paths

FIGURE 24: Overview of the microscope objective and its surrounding.

Figure 24a shows a horizontal cut through the glass cell with mounted objective and the connection to the steel cross, the external magnetic field coils, and a passive multilayer magnetic shielding. Components for controlling the magnetic fields will be discussed in the next section (§ 13). Figure 24b sketches some of the important future light paths through the glass cell. They include

- the transport beam of 532 nm, operated by translating a lens on an air-bearing mechanical stage<sup>14</sup> and optically relaying the corresponding focus shift on the atom position;
- a large-spacing vertical optical lattice formed by two crossed beams entering from the side, likely in accordion configuration, to pre-compress the sample in the vertical direction and facilitate loading into the tight vertical lattice;
- the tight vertical lattice formed by retroreflexion of a 1550 nm beam off the miniature mirror;
- the horizontal lattice, formed by two 532-nm beams in bow-tie arrangement with multiple possible polarisation configurations, running along the cell diagonals and crossing at right angle;

<sup>(</sup>b) Outline of important optica microscope.

<sup>14</sup> Model ABL 1500, Aerotech, Inc., PA/USA

- one or multiple horizontal absorption imaging paths through the side windows employing the broad, blue transitions; these paths might also be used for exciting the sample for fluorescence imaging with the microscope objective below;
- the fluorescence path through the objective and the bottom window;
- possible additional beams reflected in and out using the crown mirrors, e.g., for spin manipulation.

# 13

# MAGNETIC ENVIRONMENT

In all ultracold-atom experiments the ability to set the magnetic field in a precise manner is absolutely essential, to define quantisation axes, to control level splittings, or to tune s-wave interactions at Feshbach resonances. For magnetic atoms such as erbium and dysprosium, where magnitude as well as direction of the magnetic field are of decisive importance for the stability and behaviour of a sample, this is even more the case.

In the following, the coils for shaping magnetic fields in the glass cell will be described (§ 13.1), as well as an enclosure to shield the atoms from external, magnetic fields (§ 13.2).

# 13.1 MICROSCOPE CELL COILS

The trade-off in the design process of the microscope cell coils was between achieving a maximum flexibility in the shaping and switching of magnetic fields, blocking a minimum of opticall access, as well as constraints given by the magnetic shielding, spatially and in terms of material magnetisation.

The final design consists of a skeleton CNC-milled from a high-performance polymer<sup>1</sup>, whose pieces can be assembled around the glass cell. The coil skeleton itself is mounted on the clamp of the steel cross, to not put additional load on the glass cell. The design consists of two pairs of coils along the vertical/gravity direction (*z*), close to Helmholtz configuration, and two orthogonal pairs of coils in the horizontal plane, further from Helmholtz cofiguration due to contraints of space and optical access; see Figs 24a, 25. The slow pair of vertical coils has a larger number of windings and is intended for standard use, whereas the fast pair of vertical coils has only a few windings and is inteded for fast jumps in magnetic fields and for generation of RF radiation. The two identical horizontal coil pairs are arranged around the horizontal lattice beams (cf. Figs 23, 25). All coils except the two bottom vertical coils can be wound prior to assembly.

Even though FEM simulations (§ 13.2) indicate that the magnetisation threshold of the innermost shielding layer is not reached up to fields corresponding to more than a hundred Gauss in the cell centre, in order to avoid magnetisation effects it is certainly advisable to restrict the fields used in the cell to the few- or low tens-of-Gauss level.

Table 4 on the next page summarises the characteristics of the coils, Fig. 25 on page 125 shows the parts of the coils design and the corresponding calculated fields, field gradients and curvatures.

<sup>1</sup> PAS-PEEK GF30, glass-fibre reinforced polyether ether ketone, Faigle GmbH, Austria

|                                       | VERT. SLOW         | VERT. FAST         | HORZ.              |
|---------------------------------------|--------------------|--------------------|--------------------|
| <i>r</i> <sub>0</sub> (mm)            | 59                 | 64                 | 22.5               |
| coil separation (mm)                  | 54                 | 78                 | 98                 |
| windings                              | 56                 | 4                  | 16                 |
| centre field (G/A)                    | 8.0                | 0.5                | 0.6                |
| inductance <sup>a</sup> (F)           | $3 \times 10^{-5}$ | $2 \times 10^{-6}$ | $3 \times 10^{-6}$ |
| resistance <sup>b</sup> (m $\Omega$ ) | 700                | 55                 | 80                 |

TABLE 4: Coil specifications for the microscope chambe



FIGURE 25: Magnetic field coil system for the microscope chamber. The fields are calculated by directly integrating the Biot–Savart law for 1 A of current, respectively. The respective *local* coil symmetry axis is labelled *z*. Slow vertical bias coils (*left column*), fast vertical bias or RF coils (*middle column*) and horizontal coils (*right column*). First row: Coil geometry. Second row: Field on axis. Third row: Field gradient on axis. Fourth row: Field curvature on axis. The coil characteristics are summarised in Table 4 on the facing page.

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#### 13.2 MAGNETIC SHIELDING

For very field-sensitive measurements it becomes necessary to protect the atomic sample from magnetic stray fields of the environment, such as the earth magnetic field or fields created by electric instrumentation. The choice of protection strategy and technique mainly depends on the modulation frequency of the external field.

Low-frequency noise In laboratory practice, low-frequency noise is usually the more critical one. Its impact can be reduced either by active compensation using magnetic coils to counter the external fields,<sup>2</sup> or by enclosing the experimental chamber in a passive magnetic shielding. In principle, there are two types of passive magnetic shieldings, (i) superconducting shields at cryogenic temperature<sup>3</sup>, from which weak external fields are expelled via the Meissner–Ochsenfeldt effect (Fig. 26a), or (ii) soft-ferromagnetic shields, which rely on 'flux shunting'. Flux shunting means that the shield material has a high relative permeability and thus 'guides' the field lines around the protected volume (Fig. 26b, c). Among the most commonly used soft ferromagnetic materials for such shieldings are Mu-metal, with a relative permeability  $\mu_{\rm r} \sim 4.7 \times 10^5$  and saturation flux density  $\sim 0.75 \times 10^4$ G, and Supra-50, with a lower permeability  $\mu_{\rm r} \sim 2 \times 10^5$  but a higher saturation field  $\sim 1.5 \times 10^4$ G [90].



FIGURE 26: Field line distribution in the vicinity of a cylindrical magnetic shield. Transverse field for (a) a conductive and (b) a ferromagnetic shield. (c) Competing effects of an axial field within a cylinder. Images modified from Refs [56] (a, b) and [213] (c).

High-frequency noise For fast oscillating fields (on the order of tens of Hz and higher), eddy current cancellation ('skin effect') becomes the dominant shielding process. This effect is the strongest for good conductors such as copper, but in practice, also ferromagnetic DC shields typically provide a sufficient AC shielding [299]. The more crucial step is therefore to find a good shielding for slowly-varying fields.

<sup>2</sup> Since no probe for feedback can be put locally into the vacuum chamber, often feed-forward is applied, for example in the groups of Jean Dalibard at Collège de France, Paris.

<sup>3</sup> This strategy is followed, e.g., in the group of Gerhard Kirchmair in Innsbruck for protection of superconducting quantum bits.

Since they can operate at room temperature, ferromagnetic shields are far more often encountered in quantum gas experiments than superconducting shields (see, e. g., Refs [89, 90, 159]). Also for the Er–Dy experiment, we opted for this strategy, with the possibility to combine it with an additional, active field stabilisation.

The performance of a magnetic shield is characterised by the shielding factor

$$S = B'/B, \tag{98}$$

where B'(B) is the field at the centre point in presence (absence) of the shield. From our estimates of requirements for future microscope experiments, a shielding factor of  $\sim 10^3$  was desirable and set as a target.

#### Analytical estimates for ferromagnetic shields

In general, the shielding efficiency (98) of a ferromagnetic shield depends on (i) the relative permeability  $\mu_r$  of the shield material, (ii) the geometry of the shield, and (iii) the effect of possible holes.

Concerning the geometry, the best shielding performance would be obtained for a spherical shell, whereas for production reasons the majority of magnetic shields has a cylinder (intermediate) or box form (inferior) [299].

Concerning the effect of holes, the field entering into a tube of radius *R* along *x* decreases exponentially  $\propto e^{-\beta x/2R}$ , with  $\beta \approx 7$  for transverse and  $\approx 4.5$  for axial field, whereas for a flat surface perpendicular to *x*, the field entering through an opening drops  $\propto x^{-3}$  [299].

In regard to the shield for the Er–Dy experiment, we consider a cylinder of radius R, shell thickness  $d \ll R$ , length L and a material with  $\mu_r \gg 1$  The flux entering through the ends is exponentially suppressed and negligible for L > 2R. In this case, the residual field is completely given by the field spilling through the walls (cf. Fig. 26c).

We first consider a homogeneous, transverse field. Since flux entering through the ends is negligible, the shielding is the same as for an infinite cylinder, with the analytic solution [188]

$$S^{t} = \frac{\mu_{r}d}{2R}.$$
(99)

For axial fields, except for pathologic cases [213], the performance is improved when end caps are added to the cylinder. If they are taken into account, the axial shielding efficiency for a cylinder of radius *R*, length *L* and ratio  $\alpha = L/2R > 1$  reads

$$S^{a} = \frac{4\eta S^{t} + 1}{1 + 1/2\alpha} \quad \text{with} \quad \eta = \frac{1}{\alpha^{2} - 1} \left( \frac{\alpha}{\sqrt{\alpha^{2} - 1}} \ln \left( \alpha + \sqrt{\alpha^{2} - 1} \right) - 1 \right), \quad (100)$$

a geometry-dependent demagnetisation factor [187, 188]. The axial shielding is therefore always smaller than the transverse shielding. For short cylinders,  $\alpha \gtrsim 1$ , the axial and transverse shielding efficiency are similar in magnitude,  $S^t \gtrsim S^a$ , whereas for long cylinders, the axial shielding quickly vanishes,  $S^a \xrightarrow{\alpha \to \infty} 1$ , due to field spilling through the walls (Fig. 26c).

Shielding factor

Cylindrical shields

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Multilayer shields

The effect of a shield can be enhanced by enclosing it wholly in a bigger shield. Following a magnetic circuit approach [278, 299], the efficiency of such a nesting of *N* individual shields can be worked out analytically, with the dominant term

$$S^{\mathbf{x}} \approx S_N^{\mathbf{x}} \prod_{i=1}^{N-1} S_i^{\mathbf{x}} \left( 1 - \left( \frac{X_i}{X_{i+1}} \right)^k \right). \tag{101}$$

Here,  $X \in \{R, L\}$  is the characteristic length scale of layer *i*, and  $S_i^x \in \{S^t, S^a\}$  is the shielding factor in the corresponding direction. For cylindric shape, these are given by Eqs 99–100. The scaling exponent *k* is geometry-dependent; in good approximation k = 3 for a spherical shield, and k = 2 and k = 1 for a cylindrical shield in transverse and axial direction, respectively [299].

General guidelines

From the above reasoning and analytical estimates, we can distill some simple rules of thumb to keep in mind for the design of the shielding:

- The optimal shape is a sphere (or an approximation thereof), which provides good shielding in all directions and with the fastest scaling in size (Eq. 101).
- For a cylinder, a length-to-diameter ratio  $\sim$  1 yields the best performance.
- Since the shielding efficiency scales inversely with size (*X*) at fixed wall thickness (*d*), smaller shields are preferential (Eq. 99).
- Thin shield of multiple layers are superior to a single, thick shield 'it helps to shield the shielding'.
- Avoid discontinuities (e.g., cuts, improper welds, etc.) in the material to ensure unhindered guiding of magnetic flux.
- If discontinuities are unavoidable, e.g., for an assembly of multiple parts, the parts should have sufficiently overlap.
- Avoid openings, since they lead to flux leakage.
- If an opening is unavoidable, it can help to add a collar since the flux entering through a tube (hole) falls off exponentially (cubically).

# Shielding design and finite-element analysis

- *Large vs small* The fundamental design decision for the microscope shielding in the Er–Dy experiment was between the following options:
  - A large, box-type shield which fits the cell and surrounding close breadboards, optics, and other instrumentation within it, or
  - a small, cylindrical shield which fits only the cell such that all surrounding optics and instrumentation have to be placed outside.

Large box-type shieldings are, e.g., used in the Blatt ion laboratories in Innsbruck. Considering the points from the previous chapter, and for the greater flexibility offered by not having to put optics and instrumentation inside the shielding, the compact cylinder type was chosen.

Under the spatial constraints imposed by the geometry of vacuum apparatus and the microscope coils, a preliminary design for a four-layer magnetic shield was drafted.<sup>4</sup>

To obtain a prediction of the shielding performance beyond the general guidelines from the previous section, numerical simulations are needed. Unfortunately, applying the analytic formulae to a complex geometry to elucidate, e.g., the expected shielding performance away from the exact centre of symmetry, or the effect of openings for laser beams and cables, is a nontrivial task. A possible approach to such intricate problems is given by the finite-element method (FEM).

The idea behind a FEM simulation is to divide a system with complex geometry into a finite number of discrete, small subsystems. These small systems span a representative mesh of the whole system. The problem is then first handled locally on the level of the subsystems and their respective boundaries, which can be solved more easily, and later assembled into an approximation of the solution for the system as a whole, using techniques from the calculus of variations [243, 244].

For our study of the shielding efficiency, the Comsol<sup>®</sup> Multiphysics environment was employed, since it allows to directly import the computer-aided design (CAD) drawing and offers an automated meshing routine. Then the FEM analysis is used to simulate the magnetic field inside the shield in presence of external fields. This allowed us to verify the general expectations and to identify weaknesses of the prototype design.

In a first step, a static, homogeneous magnetic field along the different spatial directions was applied *outside* the shielding. It became apparent (i) that edges and corners can lead to flux focussing and localised high-field regions in the shield material, which may exceed the saturation threshold. Therefore, where possible, edges and corners should be rounded. By the same effect, (ii) while small circular holes in the shielding prevent fields from entering more efficiently, the hole radius has to be large enough not to lead to flux focussing. Furthermore, it turned out that (iii) the overall shielding performance along the atom transport direction was limited by the large opening for the vacuum connection.

In a second step, individual simulations were performed including the fields of the individual pairs of microscope cell coils *inside* the shielding. These simulations showed that (iv) for high coil currents in the microscope coils, the saturation threshold for Mumetal can be exceeded in nearby regions of the innermost shield.

A revised design improved on the identified weaknesses by (i–ii) adjusting roundings and diameters, (iii) reducing the opening for the vacuum connection and adding a collar to it, and (iv) changing the material of the innermost shield from Mu-metal to Supra-50, which has a sufficiently higher saturation threshold.

Subsequent simulations of the revised design, some of which are shown in Fig. 27 on the next page, have been performed to verify the improvements.

Finite-element method

Identified design weaknesses

Revised design

<sup>4</sup> We gratefully acknowledge Dimitrios Trypogeorgos, INO-CNR and Università di Trento, for helpful discussions during the design process. Cf. also Refs [89, 90].



(c)  $\vec{B} \parallel \vec{e}_z$ 

FIGURE 27: *FEM simulation results of the final shielding design.* (*Left column*) Vector fields for an applied homogeneous external magnetic field pointing in the three spatial directions (a–c): x is along the transport axis, y is perpendicular to transport direction and cylinder axis, z is along the cylinder axis. The strength of the magnetic flux inside the metal is colour-coded, increasing from blue to red. (*Right column*) Calculated magnetic flux density along the three spatial directions plotted for the same external fields.



(a) *B* on the innermost shield layer for the coil pair along the vertical axis ( $\vec{e}_z$ ). Small magenta arrows indicate flux direction.



(b) *B* along the three spatial axes ( $\vec{e}_{x,y,z}$ ); note that axes pass through shield openings.



(c) *B* on the innermost shield layer for the coil pair along the horizontal diagonal  $(\vec{e}_+)$ . Small magenta arrows indicate flux direction.



(d) *B* along the coil symmetry axis ( $\vec{e}_+$ ), perpendicular to it ( $\vec{e}_-$ ), and vertically ( $\vec{e}_z$ ); note that axes pass through shield openings.

FIGURE 28: Magnetic flux density in the innermost shielding layer for active microscope cell coils. (Top row) Flux for 10 A of current in the slow vertical coil pair. (Bottom row) Flux for 10 A of current one of the horizontal coil pairs along one of the diagonals,  $\vec{e}_{\pm} = (\vec{e}_x + \vec{e}_y)/\sqrt{2}$ .

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# The finalised shield

The revised shield design was then commissioned for manufacture by Magnetic Shields Ltd, UK. It consists of three outer shells made from Mu-metal, and the innermost made from Supra-50, all separated through milled nylon spacers. The shield features apertures for optical access both axially and radially, as well as openings for routing of cables and screw holes for attachment to the experiment. After fabrication, the shield underwent a heat treatment (4 hours at 1150 °C) for magnetic annealing. The final measured shielding factors are  $> 10^3$  both in axial and transverse direction, meeting our initial demands. The slightly better performance of the simulated shield ( $\sim 10^4$ ) is most probably due to the assembled, nested pieces being perfectly flush with each other in the CAD drawing, allowing an unhindered magnetic flux from one to another, and a correspondingly lower flux spilling to the inside. For a more conservative simulation, it could be helpful to artificially add space between the pieces to emulate this effect.

A mechanical drawing of the shield is shown in Fig. 29; a photograph of the shield ready for implementation is shown in Fig. 30.



FIGURE 29: *Drawing of an axial cut through the design* (vacuum flange entry on the left). Numbers (1)–(2) Supra-50 shells, (3)–(8) Mu-metal shells, (9)–(14) Nylon spacers. All dimensions are in millimetres.





(a) Bottom (*left*) and top half (*right*, flipped upsidedown) of the magnetic shielding.

(b) The full assembled shielding.

FIGURE 30: *The Er–Dy four-shell soft magnetic shielding*. The collar increases the shielding efficiency over the opening for the vacuum connection along the transport direction.

A LOOK BACK, AND A LOOK AHEAD

# CONCLUSION

My work in the Ferlaino group in Innsbruck can be broadly grouped into two categories, experimental research on dipolar quantum gases, and technology development for present and future experiments.

Chronologically, technology development has first concentrated on a 1-µm level resolution imaging system for our experimental main chamber, aimed at the ability to image bulk dipolar quantum gases and mixtures directly in trap. This imaging system has been designed and engineered from scratch, tested for performance, and implemented into the apparatus. It has since become an indispensable experimental standard tool for us, both in in-situ (phase-contrast) and time-of-flight (absorption) imaging mode. In phase-contrast operation, this imaging system has for the first time allowed us to directly probe the in-situ density distribution of dipolar quantum gases in an optical trap and enabled advanced studies of dipolar supersolid states.

The second big project in technology development was an extension to the existing apparatus which will allow to perform quantum gas microscopy with single-atom resolution. To this aim, a full microscope design that accommodates the needs for quantum gases of both, erbium and dysprosium, has been drafted and engineered from scratch. This includes the design of the microscope optics and associated mechanical parts, of the surrounding glass cell and other vacuum components, of the microscope magnetic coils and a multilayer passive magnetic shielding, as well as performance tests of the designed components and the developments of non-standard experimental techniques and strategies.

Scientifically, at the start of my time Innsbruck we have worked on the realisation of a two-species five-beam magneto-optical trap for erbium and dysprosium [142], which has allowed the production of the world's first dipolar quantum mixtures of erbium and dysprosium shortly after [283]. We have further investigated these quantum mixtures by means of Feshbach spectroscopy [82] and using a novel technique which directly probes the inter-species repulsion between quantum gases of erbium and dysprosium in trap.<sup>1</sup>

Besides these experiments on quantum mixtures, we have studied the physics of singlespecies dipolar quantum gases, with a particular focus on the parameter regime where, through an attractive influence of the magnetic dipole-dipole interaction, the standard mean-field description predicts a collapse. As reported in 2016 by the dysprosium experiment in Stuttgart, under certain conditions, stable quantum matter can be observed despite the instability predicted by mean-field theoy [145]. As has become clear, the responsible stabilisation mechanism is driven by quantum fluctuations, which can be incorporated into an extended form of the Gross-Pitaevskii equation through a beyond-meanfield correction [28, 178, 179, 225, 288, 289]. This stabilisation mechanism extends the phase diagram for dipolar quantum gases beyond the mean-field stability region and allows to enter novel phases of quantum matter, such as macrodroplets, incoherent droplet cystals, and supersolids. The study of the supersolid phase of dipolar quantum gases, a phase

<sup>1</sup> Claudia Politi et al., manuscript in preparation.

where global phase coherence and spatial periodicity coexist, now forms the core of this thesis.

In a first series of experiments, in a joint effort with our group's Erbium Laboratory, we have studied supersolid behaviour of quantum gases of <sup>166</sup>Er and <sup>164</sup>Dy, both theoretically and experimentally [67]. By analysis of the interference patterns after a free TOF expansion, we were able to quantify both the density modulation and the global phase coherence of our samples. We have compared our measurements to ground-state calculations based on the extended GPE, which have also allowed to construct the respective zero-temperature phase diagrams. For <sup>164</sup>Dy we have observed an experimental lifetime more than an order of magnitude longer than similar experiments performed in Pisa [279] and Stuttgart [44] at around the same time, and demonstrated for the first time the preparation of a supersolid state without a modification of scattering length, but through direct evaporation.

In a second series of experiments, we have studied this evaporative formation of the supersolid from a thermal gas of <sup>164</sup>Dy atoms in depth [272]. Theoretically, the regime at T > 0 is much more elusive and less understood than the T = 0 limit, since appropriate computational tools are lacking. Taking advantage of our newly implemented high-resolution main-chamber imaging system, and by combination of in-trap and time-of-flight imaging techniques, we have explore the formation and subsequent decay of a dipolar supersolid. We have observed that the two symmetries distinguishing the supersolid from the thermal gas are not broken simultaneously, but that the thermal gas first undergoes a transition to a state with periodic density modulation without long-range phase coherence, which is followed by a transition to a supersolid state with global phase coherence.

In a third series of experiments, we have investigated the response of a dipolar supersolid to an interaction quench that brings it strongly out of equilibrium [143]. In such outof-equilibrium density-modulated states global phase coherence is quickly lost, however, experiments have shown that by gently bringing the system to a regime where calculations predict a non-vanishing density overlap between the out-of-phase system fragments, global phase coherence can be restored. This possibility to repair the shattered supersolid indicates superfluid flow across the sample as well as an efficient dissipation mechanism for excitations. We have investigated the underlying mechanism in collaboration with theorists from Geneva, and were able to show that despite the dipolar supersolid being not stiff but rather soft, the essential behaviour of the de- and rephasing process is captured by a rigid Josephson junction array model. However, both experiment and simulation indicate that collective-mode excitations caused by the interaction quench significantly affect the phase dynamics, drawing a strong parallel to the effects of phonons in classical solids.

After these three projects, presented in the main body of this thesis, we have ventured to explore supersolidity in dipolar quantum gases beyond density-modulation along a single direction and were able to experimentally realise systems featuring two-dimensional supersolidity [207]. In a subsequent project, we have studied the excitation spectrum of such a 2D supersolid, with particular emphasis of the scissor mode, which has in the past often been employed as a probe relating to the superfluid fraction.<sup>2</sup>

<sup>2</sup> Manuscript in preparation. For background on the scissor mode in the context of dipolar quantum gases see, e.g., Refs [92, 249, 280], and references therein.

Looking back, my work in the Ferlaino group has been a challenge as well as a privilege. I have perceived it as a great opportunity to experience both, applied and fundamental research - to delve into the diverse technical aspects of developping and building new parts of a scientific apparatus, while still being able to conduct experiments at the forefront of science. Many of the achievements recounted here are unthinkable without the help and input of others I was lucky to enjoy. Some of our scientific projects have profited enormously from research collaborations; this can be seen as an attestation that much scientific advancement today is a product of both personal initiative as well as efficient team work. This is particularly true for the topic of supersolidity, which in the past has been called one of the "holy grails of condensed-matter physics" [254]. To be part of one of the teams that finally succeeded in its experimental realisation and study has been a great fortune and honour for me. This achievement would not have been possible without theory counsel from Russell Bisset, Misha Baranov and Rick van Bijnen (all Innsbruck), Thierry Giamarchi and Giacomo Morpurgo (both Geneva), Santo-Maria Roccuzzo, Alessio Recati, Giacomo Lamporesi (all Trento), Luis Santos (Hannover), Blair Blakie (Dunedin), Massimo Boninsegni (Edmonton), Philippe Chomaz (Versailles), Thomas Pohl (Aarhus), Boris Svistunov and Nikolay Prokof'ev (both Amherst), and has been stimulated enormously by the friendly competition with the experimental groups of Tilman Pfau (Stuttgart) and Giovanni Modugno (Pisa).

# OUTLOOK

The Er–Dy experiment and the team around it have been lucky to see a period of great productivity over the last years. It is my hope and my conviction that this will continue in the future.

Supersolidity has been a fascinating topic ever since it was born from pure plays of thought in the 1950s and 60s, and now, since it has finally been realised in experiment, it will certainly entertain many experimentalists and theorists for years to come. In particular, the realisation of two-dimensional supersolidity has now opened avenues very promising for complex phenomena such as quantised vortices inside a supersolid – the ultimate proof of superfluidity. Also, with a mixture apparatus as the Er–Dy machine, the entry into regimes where droplet or supersolid states are absent in single-species gases, but induced by interspecies interactions in a mixture, seem in realistic range (cf. Refs [31, 169, 268]).

An upgrade to the Er–Dy experiment that will hopefully go into operation within the next months is an additional MOT cooling step on the 841- and 741-nm transitions for erbium and dysprosium, respectively, which is currently set up by our master student Nefeli Sonnberger. These transitions have widths of  $\Gamma_{841}/2\pi \approx 8 \text{ kHz}$  [231] and  $\Gamma_{741}/2\pi \approx 1.8 \text{ kHz}$  [185], more than an order of magnitude more narrow than our standard MOT transitions (§ 5.1), offering a significantly lower Doppler temperature limit. Using this additional cooling step can enable extremely fast production of BECs in under a second as demonstrated recently at Harvard [230, 231]. Such a decrease in cycle time will enhance our experimental possibilities both in terms of higher data statistics, as well as data stability, since for significantly shorter measurement duration, the impact of longterm drifts in the experiment will be less severe.

Most significant will certainly be the integration of the quantum gas microscope into the Er–Dy apparatus, which has finally commenced (see Fig. 31) and should, according to the current time planning, hopefully be concluded during the forthcoming months. As detailed in § 10, this setup holds great promise for novel experiments with dipolar atoms on optical lattices. Combining the microscope setup with a DMD setup developed by our former master student Sandra Brandstetter [50] could further allow to project arbitrary potentials on our atomic samples. In lattice experiments, such potentials can be used for quantum state engineering (cf. Ref. [62]), whereas without the horizontal lattices, it might enable studies of homogeneous, bulk 2D quantum gases (cf. Refs [127, 305]) and theoretically predicted effects unique to dipole-dipole-interacting systems, such as the modification of vortex properties [194, 203].



(a) Front view.

(b) Top view.

FIGURE 31: Assembly of the microscope vacuum section. These photographs show the microscope objective inside the glass cell after indium sealing, initial pump-down, helium leak testing, and prior to baking.

APPENDIX

'What're quantum mechanics?' – 'I don't know. People who repair quantums, I suppose.'

– Sir Terry Pratchett, *Faust Eric* (1990)<sup>1</sup>

# A

### SOME FIELD THEORY OF QUANTUM GASES

To describe quantum many-body systems in which the particle number can vary, simple quantum mechanics is insufficient and a more advanced theory is needed. In quantum field theory (QFT), particles are interpreted as the excitations of the underlying (particle) field, and as such, they can be created from the vacuum or annihilated.

There exists a plethora of recommendable literature for QFT, including especially the book of Peskin & Schröder [222] and the lecture notes of Timo Weigand [292]. For a focus on applications to quantum gases, I can particularly recommend the lecture notes by Jook Walraven [290]. The following recapitulation of basic concepts is to large extents based on these notes, and I largely adapt their notation because I have found it the most consistent and complete.

A.1 FROM ONE ...

In quantum mechanics, a particle is described by a state vector, which in Dirac notation can be written down in representation-free form as  $|\chi\rangle$ . Such a vector is normalised by the inner product  $\langle \chi | \chi \rangle = 1$ , where the bra  $\langle \chi | = |\chi\rangle^{\dagger}$  is the Hermite conjugate of the ket.

Any measurement of the physical properties of state  $|\chi\rangle$  is connected to a hermitian operator  $\hat{O}$ . The *observable*  $\hat{O}$  defines a complete set of eigenstates  $\{|o_i\rangle\}$  which span the Hilbert space  $\mathbb{H}$  of the system and fulfil the conditions of *orthonormality* ( $\langle o_i | o_j \rangle = \delta_{ij}$ ) and *completeness* (i. e., any state  $|\chi\rangle \in \mathbb{H}$  can be written as  $|\chi\rangle = \sum_i |o_i\rangle \langle o_i | \chi\rangle$ ). Possible outcomes of measurements are represented by the spectrum of the observable, i. e. its real eigenvalues  $\hat{O} | o_i \rangle = o_i | o_i \rangle$ .

The projection of  $|\chi\rangle$  onto the eigenstate  $|o_i\rangle$  gives the probability amplitude of measuring the outcome  $o_i$  for the observable  $\hat{O}$  and is termed the *wavefunction*,  $\chi(o_i) = \langle o_i | \chi \rangle$ , in the representation defined through the basis  $\{|o_i\rangle\}$ . An example is the continuum of eigenstates  $|\vec{r}\rangle$  of the position operator  $\hat{r}$ , for which  $\chi(\vec{r}) = \langle \vec{r} | \chi \rangle$  is called the wavefunction in position space.

Reversely, from a wavefunction  $\chi(o_i)$  in a certain representation the representation-free state can be obtained by the sum  $|\chi\rangle = \sum_i \chi(o_i) |o_i\rangle$  or, for a continuous basis such as  $|\vec{r}\rangle$ , the integral  $|\chi\rangle = \int \chi(\vec{r}) d^d r |\vec{r}\rangle$ . The probability density of measuring the outcome  $o_i$  for an arbitrary state  $|\chi\rangle$  is given by the modulus-squared of the wavefunction,  $\mathscr{P}_i = |\chi(o_i)|^2 = |\langle o_i | \chi \rangle|^2$ . By the motivation that the sum of the probabilities of all possible measurement outcomes should equal unity, we obtain by Parseval's identity<sup>1</sup>

$$1 \stackrel{!}{=} \sum_{i} \mathscr{P}_{i} = \langle \chi | \chi \rangle , \qquad (102)$$

the common normalisation of quantum mechanical state vectors. The expectation value of the observable  $\hat{O}$  with respect to the state  $|\chi\rangle$  is  $\langle \hat{O} \rangle = \langle \chi | \hat{O} | \chi \rangle$ . State normalisation

Observables

Wavefunctions

<sup>1</sup> I owe credits for this quotation to Sandra Brandstetter, the biggest Pratchett fan the Ferlaino group has seen to-date.

<sup>1</sup> This identity asserts that in any Hilbert space  $\mathbb{H}$  with an inner product  $\langle \cdot | \cdot \rangle$  and an orthonormal basis  $\{|o_i\rangle\}$  for every  $|\chi\rangle \in \mathbb{H}$  holds  $\sum_i |\langle \chi | o_i \rangle |^2 = \langle \chi | \chi \rangle$ . It is the formal argument for the physical intuition that the energy of a signal in momentum space and in position space should be identical.

## IV SOME FIELD THEORY OF QUANTUM GASES

A.2 ... TO MANY

An *N*-body state

$$|X_N\rangle = |\chi_1, \dots, \chi_N\rangle \equiv |\chi_1\rangle \otimes |\chi_1\rangle \otimes \dots \otimes |\chi_N\rangle \tag{103}$$

Fock representation is an element of the direct product of the *N* single-particle Hilbert spaces  $\mathbb{H}_i$ , the *N*-body Hilbert space  $\mathbb{H}^N = \bigotimes_{i=1}^N \mathbb{H}_i$ . In Fock representation, where only the occupation numbers of the single-particle states are noted, the *N*-body state  $|X_N\rangle$  simply reads

$$|n_1, n_2, \dots, n_\ell\rangle = |\underbrace{s_1, s_1, \dots, s_\ell}_{n_1} \underbrace{s_2, s_2, \dots, \dots, s_\ell}_{n_2}\rangle \quad \text{with} \quad N = \sum_i n_i, \tag{104}$$

where usually only the occupied states are noted. For bosons the states  $|s_i\rangle$  can be multiply occupied and the order of writing down the states does not matter, for fermions all states can be occupied by one particle at maximum and, as the state has to be antisymmetrised, the order is subject to convention (e.g., ordering by energy).

#### *Particle symmetry*

Two *N*-body states must be called equivalent if the measurement of *any* physical observable  $\hat{O}$  gives the same result for both of them. The exchange operator  $\hat{K}_{ij}$  swaps the particles *i* and *j* when it acts on a ket,

$$\hat{K}_{ij}|X_N\rangle = \hat{K}_{ij}|\dots,\chi_i,\dots,\chi_j,\dots\rangle = |\dots,\chi_j,\dots,\chi_i,\dots\rangle = |X'_N\rangle.$$
(105)

If the particles *i* and *j* are indistinguishable, the new and old ket must be equivalent, so for the expectation value  $\langle \hat{O} \rangle$  must hold  $\langle X_N | \hat{O} | X_N \rangle = \langle X'_N | \hat{O} | X'_N \rangle$ . This is only possible for arbitrary  $\hat{O}$  if the exchange operator is unitary and states of *indistinguishable* particles are eigenkets of it,  $|X'_N\rangle = e^{\pi i s} |X_N\rangle$ , with initially  $s \in \mathbb{R}$ .

Of course, swapping the same particles twice must return the original state:

$$\hat{K}_{ij}\hat{K}_{ij}|X_N\rangle = |X_N\rangle \iff e^{2\pi i s} = 1,$$
(106)

so  $\hat{K}_{ij}$  must be hermitean and

$$s = \begin{cases} n & \iff |X'_N\rangle = + |X_N\rangle \quad \text{(symmetric)} \\ \frac{1}{2}(2n+1) & \iff |X'_N\rangle = - |X_N\rangle \quad \text{(antisymmetric)} \end{cases}$$
(107)

for  $n \in \mathbb{N}_0$ . Since the energy of a state is, as any observable, by definition invariant under exchange of indistinguishable particles, the Hamilton operator commutes with  $\hat{k}_{ij}$ . This makes the eigenvalue  $\pm 1$  of  $\hat{k}_{ii}$  a constant of motion, wherefore the exchange symmetry of a state is conserved in time.

By the spin–statistics theorem [218, 263] we can identify *s* with the spin of the particles. *Fermions*, antisymmetric upon exchange, have half-integer, whereas *bosons*, symmetric upon exchange, have integer spin. Assuming two fermions in the same single-particle state immediately leads to the contradiction

$$-|X_N\rangle = \hat{K}_{ij} |X_N\rangle \stackrel{(\chi_i = \chi_j)}{=} |X_N\rangle \iff |X_N\rangle = |0\rangle.$$
(108)

It follows that single-particle states can be occupied by not more than one fermion, the Pauli exclusion principle [217].

Pauli principle

Identical bosons, in contrast, are allowed to occupy the same state, and since the counting of states has to be modified compared to classical states, the probability for multi-occupancy is enhanced (cf. Fig. 1 on page 5).

# Construction operators

In many realistic quantum many-body scenarios, energy as well as particles can be exchanged with the environment. The suitable setting for such scenarios is the Grand Hilbert (or Fock) space. It is constructed by direct summation of all correctly symmetrised *N*-body Hilbert spaces,  $\mathbb{H}^{G} = \bigoplus_{N=0}^{\infty} \mathbb{H}^{N}$ .

When a particle is added to or removed from an *N*-body system, one switches within  $\mathbb{H}^{G}$  from  $\mathbb{H}^{N}$  to  $\mathbb{H}^{N\pm 1}$ , typically without mentioning it explicitly. This is achieved by application of the Hermite-conjugate construction (or ladder) operators

$$\hat{a}_{s}^{\dagger}|\ldots,n_{s},\ldots\rangle = \sqrt{n_{s}+1}|\ldots,n_{s}+1,\ldots\rangle, \qquad (109)$$

$$\hat{a}_s |\dots, n_s, \dots\rangle = \sqrt{n_s} |\dots, n_s - 1, \dots\rangle, \qquad (110)$$

which abide by the commutation (anticommutation) relations for bosons (fermions), marked below with a '-' ('+') index:

$$[\hat{a}_s, \hat{a}_\ell^{\dagger}]_{(\pm)} = \delta_{s\ell} \quad \text{and}$$

$$(111)$$

$$[\hat{a}_s^{\dagger}, \hat{a}_\ell^{\dagger}]_{(+)} = 0 - [\hat{a}_s, \hat{a}_s]_{(+)}$$

$$(112)$$

$$[\hat{a}_{s}^{\mathsf{T}}, \hat{a}_{s}^{\mathsf{T}}]_{(\pm)} = 0 = [\hat{a}_{s}, \hat{a}_{s}]_{(\pm)}.$$
(112)

From the construction operators one can define the number operator

$$\hat{n}_s = \hat{a}_s^\dagger \hat{a}_s \tag{113}$$

which counts the occupation of the respective single-particle state,

$$\hat{n}_s | \dots, n_s, \dots \rangle = n_s | \dots, n_s, \dots \rangle . \tag{114}$$

The total-number operator, which counts all particles in the system, is the sum of all number operators

$$\hat{N} = \sum_{s} \hat{n}_{s}.$$
(115)

In position representation, by inserting the closure relation  $1 = \int d^d r |\vec{r}\rangle \langle \vec{r}|$  the above (115) can be written as

$$\hat{N} = \int \mathrm{d}^d r \sum_{s,s'} \hat{a}^{\dagger}_{s'} \left\langle s' | \vec{r} \right\rangle \left\langle \vec{r} | s \right\rangle \hat{a}_s = \int \mathrm{d}^d r \, \hat{\psi}(\vec{r})^{\dagger} \hat{\psi}(\vec{r}) = \int \mathrm{d}^d r \, \hat{n}(\vec{r}), \tag{116}$$

where, writing the wavefunctions originally in an arbitrary, discrete single-particle basis  $\{|s\rangle\}$  in terms of the continuous position variable,  $\varphi_s(\vec{r}) \equiv \langle \vec{r} | s \rangle$ , we have introduced the *field operators* 

$$\hat{\psi}(\vec{r})^{\dagger} = \sum_{s} \varphi_{s}^{*}(\vec{r})\hat{a}_{s}^{\dagger} \quad \text{and} \quad \hat{\psi}(\vec{r}) = \sum_{s} \varphi_{s}(\vec{r})\hat{a}_{s}.$$
(117)

The field operators create and annihilate particles at position  $\vec{r}$ , i.e. in the one-particle case simply  $\hat{\psi}(\vec{r})^{\dagger} |0\rangle = |\vec{r}\rangle$  and  $\hat{\psi}(\vec{r}) |\vec{r}\rangle = |0\rangle$ . The field operators  $\hat{\psi}(\vec{r})^{\dagger}$  and  $\hat{\psi}(\vec{r})$  obey the same (anti-)commu-

Number operator

Construction operators

Field operators

#### VI SOME FIELD THEORY OF QUANTUM GASES

tation relations as the construction operators and reflect the (anti-)symmetry of the particles they describe:

$$[\hat{\psi}(\vec{r}), \hat{\psi}(\vec{r}')^{\dagger}]_{(\pm)} = \delta(\vec{r} - \vec{r}') \quad \text{and}$$
(118)

$$[\hat{\psi}(\vec{r})^{\dagger}, \hat{\psi}(\vec{r}')^{\dagger}]_{(\pm)} = 0 = [\hat{\psi}(\vec{r}), \hat{\psi}(\vec{r}')]_{(\pm)}.$$
(119)

Finally, it is worth to remember that in general a many-body system is rarely in a pure number state, but rather a linear combination of many number states.

#### A.3 PARTICLE CORRELATIONS

(1)

 $(\mathbf{a})$ 

The field operators allow to define different orders of correlation operators, depending on the number of particles involved [290]:

$$\hat{n}^{(1)}(\vec{r},\vec{r}') = \hat{\psi}(\vec{r})^{\dagger} \hat{\psi}(\vec{r}'), \tag{120}$$

$$\hat{n}^{(2)}(\vec{r},\vec{r}') = \hat{\psi}(\vec{r})^{\dagger} \hat{\psi}^{\dagger}(\vec{r}') \hat{\psi}(\vec{r}) \hat{\psi}(\vec{r}'), \qquad (121)$$

$$\hat{n}^{(3)}(\vec{r},\vec{r}',\vec{r}'') = \hat{\psi}(\vec{r})^{\dagger} \hat{\psi}^{\dagger}(\vec{r}') \hat{\psi}^{\dagger}(\vec{r}'') \hat{\psi}(\vec{r}') \hat{\psi}(\vec{r}') \hat{\psi}(\vec{r}'),$$
(122)

and so forth. The correlation functions for an arbitrary many-body state  $|X_N\rangle$  are given by the corresponding expectation values [32, 128, 290]

$$G^{(i)}(\vec{r},...) = \langle X_N | \hat{n}^{(i)}(\vec{r},...) | X_N \rangle.$$
(123)

The correlation functions characterise a quantum system and are well-defined irrespective of the particle interactions; however, their calculation is particularly easy for an ideal gas where the single particle states (cf. Eq. 117) are uncoupled.

Inserting the bosonic commutation relation and taking care of the proper normalisation of the many-body state using the grand-canonical partition sum (cf. § A.4 below), one can separate the parts of the density matrix corresponding to ground and excited states:

$$G^{(1)}(\vec{r},\vec{r}') = G_0^{(1)}(\vec{r},\vec{r}') + G_T^{(1)}(\vec{r},\vec{r}') = N_0 \varphi_0^*(\vec{r}) \varphi_0(\vec{r}') + \sum_{s \neq 0} \varphi_s^*(\vec{r}) \varphi_s(\vec{r}') \langle \hat{n}_s \rangle ,$$
(124)

where  $N_0 \equiv \langle X_N | \hat{n}_0 | X_N \rangle$  is the number of ground-state particles and  $\langle \hat{n}_s \rangle \equiv \langle X_N | \hat{n}_s | X_N \rangle$  is the expectation value of the occupation number operator for state *s*. For  $N_0 \gg 0$ , the system is called *condensed* (see § A.4.3).

Considering the one-body density of a three-dimensional homogeneous system of volume *V*, the relevant coordinate is  $R \equiv |\vec{r} - \vec{r}'|$  and the system eigenstates are given by plane waves,  $\varphi_{\vec{k}}(\vec{r}) = \exp(i\vec{k}\cdot\vec{r})/\sqrt{V}$ . In this case, there is an analytic expression for the one-body density matrix [290]:

$$G^{(1)}(R) = \underbrace{\frac{N_0}{V}}_{G_0^{(1)}} + \underbrace{\frac{1}{\Lambda^3} \sum_{\ell=1}^{\infty} \frac{z^\ell}{\ell^{3/2}} \exp\left(-\frac{\pi R^2}{\ell \Lambda^2}\right)}_{G_T^{(1)}} \xrightarrow{R \gg \Lambda} G_0^{(1)}, \tag{125}$$

where  $\Lambda$  is the de Broglie wavelength (1) and  $z = e^{\mu/k_BT}$  is the fugacity. What is important to note about Eq. 125 is that whereas  $G_T^{(1)}$  decays exponentially with R, the term  $G_0^{(1)}$  takes a constant value even in the thermodynamic limit  $(N, V \to \infty, N/V = \text{const.})$  and does not decrease with distance. This directly impacts the correlation length of the system, which can be defined straight from the

One-body density for the ideal Bose gas

Correlation length

one-body correlation function:

$$L^{(1)} = \int_0^\infty \frac{G^{(1)}(R)}{G^{(1)}(0)} dR.$$
(126)

For a thermal ideal gas it has the analytic form

$$L_T^{(1)} \equiv \int_0^\infty \frac{G_T^{(1)}(R)}{G_T^{(1)}(0)} dR = \frac{1}{2} \Lambda \frac{\text{Li}_1(z)}{\text{Li}_{3/2}(z)}.$$
(127)

where the polylogarithm function is defined as

$$\operatorname{Li}_{\alpha}(z) = \sum_{\ell=1}^{\infty} \frac{z^{\ell}}{\ell^{\alpha}} \quad \text{with} \quad \alpha, z \in \mathbb{C} \quad \text{and} \quad |z| < 1.$$
(128)

Away from  $\mu \to 0 - 0$ , the ratio  $\text{Li}_1(z)/\text{Li}_{3/2}(z) \approx 1$ , so the correlation length in the thermal part is of order  $\Lambda$ . In the condensate, in contrast,  $L^{(1)}$  is infinite. Such a behaviour is called off-diagonal long-range order (ODLRO).

More rigour is needed for the case of interacting bosons where the stationary free-particle states couple and lose their meaning. However, even though free-particle states can no longer be used to compute the density matrices, we shall see that off-diagonal long-range order can still be present and indicate *superfluid* behaviour [41, 220, 234]. The definition of the correlation functions (123) stays valid also in presence of interactions because the correlation operators (120–122) contain the full Hamiltonian [290]. For a uniform system, we conveniently consider the field operator in momentum representation

$$\hat{\psi}(\vec{k}) = \frac{1}{\sqrt{V}} \int e^{i\vec{k}\cdot\vec{r}} d^d r$$
(129)

and the corresponding momentum density  $\hat{n}(\vec{k}) = \hat{\psi}(\vec{k})^{\dagger}\hat{\psi}(\vec{k})$  with the expectation value  $n(\vec{k}) = \langle X_N | \hat{n}(\vec{k}) | X_N \rangle$ . The (spatially averaged) one-body correlation function in this case reads

$$G^{(1)}(\vec{r}) \equiv \frac{1}{V} \int n(\vec{r}', \vec{r}' + \vec{r}) \mathrm{d}^d r'.$$
(130)

From here, a straight-forward calculation reveals

$$n(\vec{k}) = \int G^{(1)}(\vec{r}) e^{i\vec{k}\cdot\vec{r}} d^d r = \mathscr{F}\{G^{(1)}(\vec{r})\}_{\vec{k}}.$$
(131)

This Fourier relation immediately implies that if there is a component in  $G^{(1)}(\vec{r})$  that does not decrease to zero for  $|\vec{r}| \rightarrow \infty$ , there has to be a  $\delta$ -like peak in  $n(\vec{k})$  and vice versa. Physically, this means that ODLRO is connected to the macroscopic occupation of a state, viz., a phenomenon like Bose–Einstein condensation, irrespective of the particle interaction. This is the fundamental connection between the superfluidity of helium-II, a strongly interacting system, and the BEC of a non-interacting ideal gas.

#### A.4 CONNECTIONS TO THERMODYNAMICS

In the following, let us take a brief look at the implications which quantum statistics have on the thermodynamic properties of a system of particles.

Density matrix for interacting systems

# VIII SOME FIELD THEORY OF QUANTUM GASES

#### A.4.1 The partition function for the grand canonical ensemble

In a grand-canonical ensemble, the system (S) under study is in contact with a large reservoir (B, 'bath') of temperature *T* and chemical potential  $\mu$ . S and B can exchange heat as well as particles, which for now are assumed non-interacting. The system S therefore has neither a fixed energy nor a fixed particle number, but we can ask for the *probability* P(E, N) of S having an energy *E* and containing *N* particles. This can be formalised by defining  $\Omega_S$ ,  $\Omega_B$ , and  $\Omega$  as the numbers of possible particle configurations within S, B, and the (closed) combined system  $S \cup B$ , respectively. If we denote the combined system's total energy by  $E_{\Omega}$ , the total particle number by  $N_{\Omega}$  and the inverse thermal energy by  $\beta = 1/k_BT$ , we can write

$$P(E,N) = \frac{\Omega_{\rm S}(E,N)}{\Omega(E_{\Omega},N_{\Omega})} = \frac{\Omega_{\rm B}(E_{\Omega}-E,N_{\Omega}-N)}{\Omega(E_{\Omega},N_{\Omega})}$$
(132)

$$\approx \frac{\Omega_{\rm B}(E_{\Omega}, N_{\Omega})}{\Omega(E_{\Omega}, N_{\Omega})} e^{-\beta(E-\mu N)} \equiv \mathcal{Z}_{\rm g}^{-1} e^{-\beta(E-\mu N)}$$
(133)

where from the first to the second line, exploiting  $E_{\Omega} \gg E$ ,  $N_{\Omega} \gg N$  we have Taylor-expanded  $\ln \Omega_{\rm B}$  up to first order in *E* and *N*, inserted the thermodynamic derivatives  $\partial_E \ln \Omega_{\rm B} = \beta$  and  $\partial_N \ln \Omega_B = -\beta \mu$ , and taken the exponential.<sup>2</sup> In the last step we have defined the grand partition function<sup>3</sup>

$$\mathcal{Z}_{g} = \frac{\Omega(E_{\Omega}, N_{\Omega})}{\Omega_{B}(E_{\Omega}, N_{\Omega})},$$
(134)

which is fixed by the normalisation

$$1 = \sum_{i} P(E_i, N_i) = \mathcal{Z}_g^{-1} \sum_{i} e^{-\beta(E_i - \mu N_i)} \qquad \Longleftrightarrow \qquad \mathcal{Z}_g = \sum_{i} e^{-\beta(E_i - \mu N_i)}$$
(135)

where the summation index *i* runs over all possible energies and particle numbers.

For a quantum gas, the partition function has to include all properly (anti-)symmetrised states. If the gas is ideal, the result factorises and we can write in Fock (number-state) representation

$$\mathcal{Z}_{g} = \sum_{n_{1}} \sum_{n_{2}} \cdots \langle n_{1}, n_{2}, \dots | e^{-\beta(\hat{H} - \mu\hat{N})} | n_{1}, n_{2}, \dots \rangle$$
(136)

$$=\sum_{n_1}\sum_{n_2}\cdots e^{-\beta\left(n_1(\varepsilon_1-\mu)+n_2(\varepsilon_1-\mu)+\cdots\right)}=\prod_s\sum_{n_s}e^{-\beta n_s(\varepsilon_s-\mu)}.$$
(137)

For bosons, the number of particles per state is unrestricted, so we have to sum from  $n_s = 0$  to  $\infty$  and recognise the geometrical series. For fermions, each state can be occupied by one particle at maximum, hence the sum runs from  $n_s = 0$  to 1. We obtain

$$\mathcal{Z}_{g} = \begin{cases} \prod_{s} \frac{1}{1 - e^{-\beta(\varepsilon_{s} - \mu)}} & \text{(bosons),} \\ \prod_{s} 1 + e^{-\beta(\varepsilon_{s} - \mu)} & \text{(fermions),} \end{cases}$$
(138)

which can be written compactly as

$$\ln \mathcal{Z}_{g} = \pm \sum_{s} \ln(1 \pm e^{-\beta(\varepsilon_{s} - \mu)}), \tag{139}$$

where Bose–Einstein (–) and Fermi–Dirac (+) statistics are distinguished by the  $\pm$  sign.

<sup>2</sup> See, e.g., Ref. [100, § 22], or the reader's favourite statistical physics textbook.

<sup>3</sup> or zustandssumme (German, 'sum over states')

The thermodynamics of the grand-canonical ensemble is governed by the grand potential<sup>4</sup>

$$\Phi_{g} = E - TS - \mu N \tag{140}$$

where *E* is the internal energy of the system, *S* is the entropy and *N* is the particle number. By calculating the total differential of  $\ln Z_g$  and comparison to Eq. 140 one finds the important identity [100]

$$\Phi_{\rm g} = -k_{\rm B}T\ln\mathcal{Z}_{\rm g},\tag{141}$$

by which all thermodynamic quantities depending on the grand potential can be related to the partition function. This is a profound result. We have started by calculating  $Z_g$  from the microstate energies  $\varepsilon_s$  (which depend, e.g., on the particle mass), and by the derivatives of the partition functions via Eq. 141 we can now relate these *microscopic* details to *macroscopic* thermodynamic quantities. In particular, the total number of bosons (–) or fermions (+) in the system is

$$N = -\frac{\partial \Phi_{g}}{\partial \mu} = k_{B}T\frac{\partial}{\partial \mu}\ln \mathcal{Z}_{g} = \sum_{s} \frac{1}{e^{\beta(\varepsilon_{s}-\mu)} \pm 1}.$$
(142)

We can thus write for the average occupation number of state  $|s\rangle$ 

$$\bar{n}_s = \frac{1}{\mathrm{e}^{\beta(\varepsilon_s - \mu)} \pm 1}.\tag{143}$$

For fermions (+), this expression is valid without restrictions and  $\bar{n}_s < 1$  for any finite temperature. For bosons (-), in contrast, there is an important caveat. Even for a finite particle number N, the sum on the r.h.s. of Eq. 142 would diverge at  $\mu \rightarrow \varepsilon_s$  for any  $\varepsilon_s$ . Therefore,  $\mu$  has to be smaller than the lowest energy,  $\min_s(\varepsilon_s) = \varepsilon_0$ . Choosing the zero of the energy such that  $\varepsilon_0 = 0$ , it follows that for ideal bosons  $\mu \leq 0$ .

#### A.4.2 The continuum approximation

For  $\mu < 0$  and an average energy large compared to the level spacing,  $k_{\rm B}T \gg \varepsilon_1$ , we can approximate the discrete energy spectrum of the ideal gas as continuous.<sup>5</sup> In this continuum approximation, the sum (142) can be evaluated as an integral,

$$\sum_{s} \rightarrow \frac{1}{(2\pi\hbar)^{d}} \int d^{d}p \, d^{d}r \quad \text{and} \quad \varepsilon_{s} \rightarrow E(\vec{r}, \vec{p}) = \frac{p^{2}}{2m} + U(\vec{r}), \tag{144}$$

which depends on the dimensionality *d* of the system and the shape of the trapping potential  $U(\vec{r})$ . For  $0 < z \le 1$  we can rewrite and expand the integrand in terms of the fugacity,<sup>6</sup>

$$\frac{1}{z^{-1}\mathrm{e}^{\beta E(\vec{r},\vec{p})} \pm 1} = \frac{z\,\mathrm{e}^{-\beta E(\vec{r},\vec{p})}}{1 \pm z\,\mathrm{e}^{-\beta E(\vec{r},\vec{p})}} = \sum_{\ell=1}^{\infty} (\mp 1)^{\ell+1} z^{\ell} \mathrm{e}^{-\ell\beta E(\vec{r},\vec{p})}.$$
(145)

Note that for  $k_{\rm B}T \gg |\mu|$  the fugacity *z* is a small number and the first term in the sum dominates. In this case, we recover the classical Maxwell-Boltzmann distribution,  $\propto \exp(-E/k_{\rm B}T)$ , irrespective of the particle symmetry.

<sup>4</sup> We omit the explicit notation of the averages,  $E \equiv \langle E \rangle$  and  $N \equiv \langle N \rangle$ , which reflect that heat and particles can be exchanged with the reservoir.

<sup>5</sup> This is also called the Thomas–Fermi approximation.

<sup>6</sup> The case z > 1 is only relevant for strongly degenerate Fermi gases, which need a different treatment; see, e.g., Ref. [290, § 10].

## X Some field theory of quantum gases

In the following, we will restrict our analysis to the bosonic case ('-' on the l.h.s. of Eq. 145); treatments of Fermi systems can be found elsewhere [108, 150, 290]. As motivated in § 1.2, we have to keep in mind that for bosons the ground-state occupation can diverge for  $\mu \rightarrow 0$  and it is safer to treat it separately from the thermal states,  $N = N_0 + N_T$ . The continuum approximation directly gives the number of available thermal states,

$$N_T = \frac{1}{(2\pi\hbar)^d} \sum_{\ell=1}^{\infty} z^\ell \int e^{-\ell\beta E(\vec{r},\vec{p})} d^d p \, d^d r.$$
(146)

Let us recall the density of states in the system,

$$g(\varepsilon) \equiv \frac{1}{(2\pi\hbar)^d} \int \delta(\varepsilon - E(\vec{r}, \vec{p})) d^d p d^d r, \qquad (147)$$

which depends on the trapping potential  $U(\vec{r})$ . For the most relevant experimental applications,  $U(\vec{r})$  has a power-law dependence on distance. For a spherical trap,<sup>7</sup> this can be written as

$$U(\vec{r}) = U_0 \left(\frac{r}{L}\right)^{d/\gamma},\tag{148}$$

with a trap strength  $U_0$ , a characteristic trap size L and a trap parameter  $\gamma$ . For example, a harmonic trap of frequency  $2\pi\omega$  has  $\gamma = d/2$  and  $U_0L^{-d/\gamma} = \frac{1}{2}m\omega^2$  for particles of mass m, whereas a box trap of length L and depth  $U_0$  has  $\gamma \to 0$ . In power-law traps, the density of states can be written as

$$g(\varepsilon) = C_{\alpha}\varepsilon^{\alpha-1}$$
 with  $\alpha = \frac{d}{2} + \gamma$  (149)

and a constant  $C_{\alpha}$  (see Table 5 and Refs [223, 290]). With this, we can bring Eq. 146 into the form

$$N_T(\mu, T) = \sum_{\ell=1}^{\infty} z^\ell \int e^{-\ell\beta\varepsilon} g(\varepsilon) d\varepsilon.$$
(150)

TABLE 5: Parameters of the density of state  $g(\varepsilon) = C_{\alpha}\varepsilon^{\alpha-1}$  for a system of *d* dimensions and a power-law trap of characteristic size *L* and trap parameter  $\gamma$ .

| TRAP        | $\gamma$      | α             | C <sub>α</sub>                                                |   |
|-------------|---------------|---------------|---------------------------------------------------------------|---|
| square well | 0             | $\frac{d}{2}$ | $\left(\frac{L}{2\pi\hbar}\right)^d \frac{S_d}{2} (2m)^{d/2}$ | a |
| harmonic    | $\frac{d}{2}$ | d             | $\left((d-1)!\prod_{i=1}^{d}(\hbar\omega_{i}) ight)^{-1}$     | b |

<sup>a</sup>  $S_d$  is the surface of the *d*-dimensional unit sphere:  $S_d = 1, 2\pi, 4\pi$  for d = 1, 2, 3.

<sup>b</sup>  $C_{\alpha}$  is given in the approximation for  $k_{\rm B}T \gg \hbar\omega$ , i. e., treating  $\hbar\omega_i n_i$  as continuous and neglecting zero-point motion [223].

With help of the polylogarithm function (128), the following relation (151) and the definition of the Euler Gamma function (152),

$$\int_{0}^{\infty} e^{-\ell x^{n}} x^{m} dx = \ell^{-(m+1)/n} \int_{0}^{\infty} e^{-x^{n}} x^{m} dx,$$
(151)

$$\Gamma(s) = \int_0^\infty e^{-x} x^{s-1} dx, \qquad s \in \mathbb{C},$$
(152)

7 The case can easily be generalised to non-isotropic orthogonal power-law traps [12].

we can evaluate the integral (150) to

$$N_T(\mu, T) = (k_{\rm B}T)^{\alpha} C_{\alpha} \Gamma(\alpha) {\rm Li}_{\alpha}(z).$$
(153)

Let us now imagine a thermal system with *N* particles in a volume *V* at a finite temperature *T* (where  $k_{\rm B}T \gg |\mu|$ ). If we keep *T* constant and keep adding particles to *V*, we have to distribute them into thermal states and thus also  $N_T$  should increase. The polylogarithm  ${\rm Li}_{\alpha}(z)$  grows monotonously with the fugacity  $z = e^{\mu/k_{\rm B}T}$ . This means when we add particles, the chemical potential has to increase (but cannot exceed zero).

The implications now differ greatly for the cases  $\alpha > 1$  and  $\alpha \le 1$ .

#### A.4.3 Bose–Einstein condensation of the ideal gas

For  $\alpha > 1$ , there is a convergence limit

$$\lim_{z \to 1-0} \operatorname{Li}_{\alpha}(z) = \operatorname{finite} = \zeta(\alpha) \tag{154}$$

which can be written in terms of the Riemann zeta function,  $\zeta(s) = \sum_{\ell=1}^{\infty} \ell^{-s}$  with  $s \in \mathbb{C}$ . This means that there is only a finite number  $N_T$  of thermal states even for  $\mu \to 0$ . Since no thermal states are available, additional particles have to go into the ground state, and the value of  $\mu$  is fixed by the requirement that  $N = N_0 + N_T$ , i.e.

$$N_0 = \frac{z}{1-z} \quad \Longleftrightarrow \quad \mu = -k_{\rm B}T\ln\left(1 + \frac{1}{N - N_T}\right) \approx -\frac{k_{\rm B}T}{N - N_T}.$$
(155)

Furthermore, we can define a critical temperature  $T_c$  from the convergence limit of Eq. 153 for a total particle number  $N = N_T$ ,

$$k_{\rm B}T_{\rm c} = \left(\frac{N}{C_{\alpha}\Gamma(\alpha)\zeta(\alpha)}\right)^{1/\alpha}.$$
(156)

For the case of a 3D harmonically trapped gas of  $10^6$  particles this approximates to  $k_B T_c \approx N^{1/3} \hbar \omega$ , so the critical temperature corresponds to about hundred times the harmonic oscillator spacing. This shows that for temperatures not too much below  $T_c$  the requirement for the continuity approximation (144) is still fulfilled,  $|\mu| \ll \varepsilon_1 \ll k_B T$ ; the condensation happens at *finite temperature*. In Maxwell–Boltzmann statistics, in contrast, macroscopic occupation of the ground state can only occur at  $k_B T \ll \varepsilon_1$ .

Rearranging Eq. 156 and inserting back into Eq. 153 directly yields the dependence of condensate fraction on temperature for  $T \leq T_c$ :

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^{\alpha} \longrightarrow \begin{cases} 1 & \text{for } T \to 0\\ 0 & \text{for } T \to T_c \end{cases}$$
(157)

#### A.4.4 Degenerate Bose gases in lower dimensions

For  $\alpha \leq 1$ , in contrast, the polylogarithm diverges for  $z \to 1-0$  (i. e.  $\mu \to 0-0$ ), so also N' diverges and we can accommodate arbitrarily many particles in thermal states. Hence, for  $\alpha \leq 1$  a Bose gas can be come degenerate ( $n\Lambda^d > 1$ ) *without* particles being forced to occupy the single-particle ground state.

Consequently, if one *assumes* condensation in a system with  $\alpha \leq 1$ , it was first shown using

Mermin–Wagner– Hohenberg theorem

Critical Temperature equivalent spin models<sup>8</sup> that the energy of the Nambu–Goldstone modes associated to this symmetry breaking tends to zero for large enough systems [69, 137, 195, 232]. It is therefore inevitable that large-wavelength fluctuations are thermally excited for  $T \neq 0$  and destroy true long-range order (LRO).

The 2D case marks an important limit for bosonic systems. As argued above, for perfect homogeneity ( $\gamma = 0$ ), no macroscopic ground-state occupation can be obtained at T > 0.

BKT phase transition

Non-uniform

1D

BECs in 2D and

On the one hand, however, even though the Mermin–Wagner–Hohenberg theorem prevents a spontaneous symmetry breaking and the establishment of a spatially uniform order parameter (i. e., LRO), in uniform 2D (but not 1D) systems there can be a topological phase transition of Berezinskii–Kosterlitz–Thouless-type. Here, above a critical temperature, the correlation function  $G^{(1)}(R)$  between two points in the system at distance *R* decays exponentially as for a usual thermal gas (cf. Eq. 125), whereas below the transition temperature  $G^{(1)}(R)$  decays only algebraically with *R* [24, 25, 29, 128, 158]. The microscopic origin of this so-called quasi-long-range order is the binding of free vortices and antivortices into tight vortex-antivortex pairs.

On the other hand, if a 2D system departs from perfect uniformity by even the slightest powerlaw potential ( $\gamma > 0$ ), the limit (154) exists and a BEC can be observed at finite temperature [99]. Similarly, for 1D systems, Bose–Einstein condensation can occur for  $\gamma > \frac{3}{2}$  (cf. Ref. [224]).

<sup>8</sup> For many types of quantum gases there is a corresponding spin model of the same universality class, such as the spin XY model for the 2D Bose gas. If two different systems belong to the same universality class, their behaviour in the vicinity of a phase transition is, in many ways, similar [29, 30].

# SELECTED CONCEPTS IN OPTICS

In this chapter, some fundamental theory of the propagation is recapitulated with regard to its application for imaging in general, and imaging of quantum gases in particular. In the course of this thesis, this has been important for the development of both the high-resolution vertical imaging objective presented in § 5.2, as well as the quantum gas microscope presented in § 10. The following review is to large extents based on Refs. [42, 112, 193, 253, 269, 277].

#### **B.1 THE SCALAR FIELD APPROXIMATION**

Most generally, light propagation is described by Maxwell's equations [112, 117]. From those, one can for the case of a linear, isotropic, homogeneous, nondispersive medium derive identical vector wave equations for the electric and magnetic field  $\vec{E}(\vec{r},t)$ ,  $\vec{B}(\vec{r},t)$ , respectively. Since, under these conditions, all of their vector components evolve identically, a single, scalar wave equation is sufficient to describe the complete system:

$$\nabla^2 u(\vec{r},t) - \left(\frac{n_{\rm r}}{c}\right)^2 \frac{\partial^2}{\partial t^2} u(\vec{r},t) = 0 \tag{158}$$

Each of the vectorial components of  $\vec{E}$  and  $\vec{B}$  must abide by the scalar equation, and can hence be written in the form

$$u(\vec{r},t) = \Re\{U(\vec{r})\}e^{i\omega t},\tag{159}$$

with the position-dependent part  $U(\vec{r}) \propto e^{i\varphi(\vec{r})}$  in complex-field notation and a position-dependent phase  $\varphi(\vec{r})$ . Inserting the ansatz (159) into the scalar equation (158), it follows that the space-dependent part obeys the Helmholtz equation

$$(\nabla^2 + k^2)U(\vec{r}) = 0 \tag{160}$$

with the wave number  $k = 2\pi/\lambda$  and wavelength  $\lambda$ . In a medium with the real part of the refractive index  $n_r$ , the wavelength fulfils  $c/n_r = \lambda v$ , where *c* is the vacuum speed of light and  $v = \omega/2\pi$  is the frequency.

*Diffraction phenomena* are all deviations of light rays from straight paths that cannot be attributed *Diff* to refraction or reflexion [273]. The scalar field approximation remains applicable for diffraction phenomena if two conditions are fulfilled [112]:

- 1. The diffracting aperture is much larger than the wavelength.
- 2. The diffracted field must be observed at a distance sufficiently far from the aperture.

For the majority of applications of optical imaging, these two criteria are met. One has to be aware, however, that for systems using very high numerical apertures (NA), vector field properties can become important and the scalar equation (158) is not necessarily sufficient anymore [9, 247].

#### B.2 LIGHT FIELD PROPAGATION

In the following, let us recapitulate how light fields can be propagated between different observation planes.

Wave equation

Helmholtz equation

Diffraction

#### XIV SELECTED CONCEPTS IN OPTICS

#### B.2.1 Propagation in free space

A priori, it may not be completely intuitive how a given, complicated (monochromatic) field distribution U(x, y, 0) will propagate in space, say, along *z*. This problem becomes tractable if we interpret the complicated field U(x, y, 0) as a sum of plane waves, which we know how to propagate:

$$U(x, y, 0; t) = \sum_{i} A_{i} w_{i}(x, y, 0; t)$$
(161)

with complex amplitudes  $A_i$  and plane waves  $w_i$ . Plane waves are solutions of the Helmholtz equations. In free space, they propagate as

$$w_i(x, y, z; t) = e^{i(\vec{k}_i \cdot \vec{r} - \omega_i t)} = e^{i(k_{ix}x + k_{iy}y)} e^{-i\omega_i t} e^{ik_{iz}z} = w_i(x, y, 0; t) e^{ik_{iz}z},$$
(162)

with the angular frequency  $\omega_i = 2\pi c/\lambda_i$ , wavelength  $\lambda_i$ , and the wavenumber  $k_i = |\vec{k}_i| = 2\pi/\lambda_i$ . By  $k_i^2 = k_{ix}^2 + k_{iy}^2 + k_{iz}^2$  the three wave vector components are interrelated and we can write

$$k_{iz} = \pm \sqrt{k_i^2 - k_{ix}^2 - k_{iy}^2} = \pm 2\pi \sqrt{\lambda_i^{-2} - u_i^2 - v_i^2},$$
(163)

introducing the spatial frequencies  $u_i = k_{ix}/2\pi$ ,  $v_i = k_{iy}/2\pi$ .

We see that there are two solutions for  $k_{iz}$ , one positive, one negative, corresponding to propagation in forward and backward direction, respectively. We will restrict the following discussion to the forward propagating waves (+), but keep in mind that, since plane waves are eigenfunctions of homogeneous media, inhomogeneities in the medium can scatter waves between different plane-wave states, thus leading, e.g., to light reflexion [277].

Furthermore, we consider only  $u_i$ ,  $v_i$  fulfilling  $u_i^2 + v_i^2 \le \lambda_i^{-2}$ , which ensure that the solutions for  $k_{iz}$  are real-valued. Otherwise, imaginary solutions lead to an exponential damping of the wave over distance z, according to Eq. 162. Such a damped wave does not carry energy and is called evanescent. It is also worthwhile to note that this presents a fundamental diffraction limit for imaging: all details in the z = 0 plane with a length scale smaller than  $\lambda$  are completely damped out during propagation along z.

From the forward-propagating plane waves  $w_i$  one can, in analogy to Eq. 161, write the propagated field distribution as

$$U(x, y, z; t) = \sum_{i} A_{i} w_{i}(x, y, z; t) = \sum_{i} A_{i} w_{i}(x, y, 0; t) e^{2\pi i z \sqrt{\lambda_{i}^{-2} - u_{i}^{2} - v_{i}^{2}}}.$$
(164)

In the general case, of course, the number of contributing plane waves is not restricted and we can make  $\lambda$  (and hence k, u, v) a continuous variable, label the plane waves by u, v instead of the index *i*, and replace the sum by an integral. Dropping the explicit time dependence, Eq. 164 then takes the form

$$U(x,y,z) = \iint_{-\infty}^{+\infty} A(u,v,0) e^{2\pi i (ux+vy)} e^{2\pi i z \sqrt{\lambda^{-2} - u^2 - v^2}} du dv.$$
(165)

Angular spectrum

The complex amplitudes A(u, v) are called the *angular spectrum* and can be derived from U(x, y, 0) via Fourier transformation:

$$A(u,v,0) = \mathscr{F}\{U(x,y,0)\}_{u,v} = \iint_{-\infty}^{+\infty} U(x,y,0) e^{-2\pi i (ux+vy)} dx dy$$
(166)
By recognising that Eq. 165 in turn corresponds to an inverse Fourier transform and defining the free-space *transfer function* 

$$H(u,v;z) \equiv \exp\left(2\pi i z \sqrt{\lambda^{-2} - u^2 - v^2}\right),\tag{167}$$

we can write the propagated field very compactly as

$$U(x,y,z) = \iint_{-\infty}^{+\infty} U(x,y,0) e^{2\pi i (ux+vy)} H(u,v,z) du dv$$
(168)

$$=\mathscr{F}^{-1}\{\mathscr{F}\{U(x,y,0)\}_{u,v}H(u,v;z)\}_{x,y}.$$
(169)

So, by construction, in this frequency-domain picture the image-plane field is the sum of the propagated plane-wave components of the object-plane field.

There is an alternative form of Eq. 168 which can be obtained by application of the convolution theorem<sup>1</sup> to the right-hand side:

$$U(x, y, z) = U(x, y, 0) * h(x, y, z)$$
(170)

$$= \iint_{-\infty}^{+\infty} U(x',y',0) h(x-x',y-y';z) dx' dy'$$
(171)

The convolution kernel

$$h(x, y, z) = \mathscr{F}^{-1}\{H(u, v, z)\}_{x, y},$$
(172)

acts as a Green's function for the diffraction problem and is called the *impulse response* of the imaging system [112]. Intuitively, the impulse response is the image produced for a delta function input,  $\delta(x', y', 0)$ . Therefore, the total object field U(x', y', 0) can be interpreted as a sum of (delta) points, each creating an impulse response. The sum of all impulse responses then forms the image. This space-domain interpretation of image formation is complementary to the frequency-domain interpretation presented above.



FIGURE 32: Angular spectrum propagation. A random field E(x, 0) (*left panel*) is propagated in the positive *z*-direction (*right panel*). Note that the propagated intensity profile is constant over *z* since all plane waves are infinitely extended.

Frequency-domain picture

Space-domain picture

<sup>1</sup> For \* denoting the convolution operation,  $\{f * g\}(x) = \int f(x')g(x' - x)dx'$ , and the Fourier transforms  $F = \mathscr{F}\{f(x)\}_{\nu}$  and  $G = \mathscr{F}\{g(x)\}_{\nu}$  it holds  $\{f * g\}(x) = \mathscr{F}^{-1}\{F(\nu)G(\nu)\}_{x}$ .

## XVI SELECTED CONCEPTS IN OPTICS

## B.2.2 Propagation through pupils

In the above treatment, fields U(x, y, z) have been infinitely extended in the lateral *xy*-direction. However, in practice one is typically *not* concerned with infinitely extended plane waves, but light coming from a localised area  $\Sigma$ , say, the entrance or exit pupil of an optical system (cf. Fig. 33). This makes a big difference, since in this case not all propagated plane waves will contribute everywhere in the image plane.



FIGURE 33: *Rays, exit pupil and aperture stop of a simple imaging system with magnification* ×2. The element which limits the size of the light cone converging onto the image is called the aperture stop. Solid lines depict the chief rays (pass through centre of aperture stop) and marginal rays (pass by edges of aperture stop). The exit pupil is the geometric image of the aperture stop as it would be seen from the final image plane. The marginal ray angle  $\alpha$  ( $\alpha'$ ) sets the object (image) space numerical aperture NA =  $n_r \sin \alpha$  (NA' =  $n_r \sin \alpha'$ ) of the system, where  $n_r$  is the refractive index of the medium around the lenses. Cf. Refs [193, 269].

Historically, this problem was solved using the Huygens–Fresnel principle, where, somewhat similar to the space-domain approach (171) every point inside the aperture is interpreted as the source of a elementary, spherical wavelet. All spherical wavelets emanating from the aperture then superpose and form the total light field:

$$U(x,y,z) = \mathscr{A} \iint_{\Sigma} U(x',y',0) \frac{1}{r^2} e^{ikr} dx' dy'$$
(173)

with radii given by  $r = \sqrt{z^2 + (x - x')^2 + (y - y')^2}$ . However, the historic approach needs an empirically introduced prefactor  $\mathscr{A}$  to match experimental observations [112]. We choose to start from our frequency-domain approach (165) in the previous section, and by doing so, the prefactor will naturally arise.

The light field transmitted through the aperture  $\Sigma$  is obtained by multiplication of a pupil function *P*,

$$U_{\Sigma}(x,y) = P(x,y)U(x,y,0) \quad \text{with}$$
(174)

$$P(x,y) = \begin{cases} 1, & \text{if } (x,y) \text{ lies within } \Sigma \\ 0, & \text{otherwise.} \end{cases}$$
(175)

We use Eq. 168 to propagate  $U_{\Sigma}(x, y)$  to a plane at distance z,

$$U(x, y, z) = \mathscr{F}^{-1} \left\{ \mathscr{F} \{ U_{\Sigma}(x, y) \}_{u, v} H(u, v; z) \right\}.$$
(176)

Paraxial approximation

We will now subject  $H(u, v; z) = \exp(ik_z z)$  to the paraxial approximation. To this aim, denoting

 $\kappa_x = k_x/k = \lambda u$  and  $\kappa_y = k_y/k = \lambda v$ , we expand the forward-propagating solution (163) up to first order,

$$k_z = k\sqrt{1 - \kappa_x^2 - \kappa_y^2} \approx k\left(1 - \frac{\kappa_x^2}{2} - \frac{\kappa_y^2}{2}\right) = k - \pi\lambda(u^2 + v^2),\tag{177}$$

effectively approximating spherical wavelets by parabolic surfaces. This approximation is justified when the distance *z* between aperture and observation screen is larger than the characteristic size of the aperture,  $\emptyset$ .

We can now define the paraxial transfer function as

$$G(u,v;z) \equiv \exp\left(\mathrm{i}kz - \mathrm{i}\pi\lambda z(u^2 + v^2)\right). \tag{178}$$

With this, Eq. 176 becomes the Fresnel integral

$$U(x, y, z) = \mathscr{F}^{-1} \{ \mathscr{F} \{ U_{\Sigma}(x, y) \}_{u,v} G(u, v; z) \}_{x,y}.$$
(179)

In analogy to Eq. 170, we can write the Fresnel integral as a convolution,

$$U(x,y,z) = U_{\Sigma}(x,y) * g(x,y;z)$$
(180)

$$= \iint_{-\infty}^{+\infty} U_{\Sigma}(x',y')g(x-x',y-y';z)dx'dy',$$
(181)

where the explicit form of the paraxial impulse response

$$g(x,y;z) = \mathscr{F}^{-1}\{G(u,v;z)\}_{x,y} = \frac{1}{i\lambda z} e^{ikz} e^{\frac{ik}{2z}(x^2 + y^2)}$$
(182)

can be found using the identity<sup>2</sup>

$$\mathscr{F}^{-1}\left\{e^{-i\pi z\lambda(u^{2}+v^{2})}\right\}_{x,y} = \frac{1}{i\lambda z}e^{\frac{ik}{2z}(x^{2}+y^{2})},$$
(183)

which yields the coefficient  $\mathscr{A} = 1/i\lambda z$  mentioned in the beginning.

In the so-called far-field, where  $z \gg \beta^2 / \lambda$ , the field propagation can be simplified even further. *Fraunhofer Starting* from the paraxial impulse response in the convolution integral (181), *Fraunhofer integral* 

$$g(x - x', y - y') = \frac{1}{i\lambda z} e^{ikz} e^{\frac{ik}{2z} \left( (x - x')^2 + (y - y')^2 \right)}$$
(184)

$$\approx \frac{1}{i\lambda z} e^{ikz} e^{\frac{ik}{2z}(x^2 + y^2)} e^{-\frac{ik}{z}(xx' + yy')},$$
(185)

where terms quadratic in the pupil coordinates (x', y') have be neglected compared to screen coordinates (x, y) since the diffraction pattern in the far field is much larger than the pupil. The diffraction integral (181) then becomes

$$U(x,y,z) = \frac{e^{ikz}}{i\lambda z} e^{\frac{ik}{2z}(x^2+y^2)} \iint_{-\infty}^{+\infty} U_{\Sigma}(x',y') e^{-\frac{ik}{z}(xx'+yy')} dx' dy'$$
(186)

$$= \frac{\mathrm{e}^{\mathrm{i}kz}}{\mathrm{i}\lambda z} \mathrm{e}^{\frac{\mathrm{i}k}{2z}(x^2+y^2)} \mathscr{F}\left\{U_{\Sigma}(x',y')\right\}_{\frac{x}{\lambda z},\frac{y}{\lambda z}},\tag{187}$$

Fresnel integral

<sup>2</sup> This identity can be derived via the Fourier transform formula of a Gaussian, using the substitution  $z \rightarrow z - i\beta$  with  $\beta > 0$  such that the integral converges, and letting  $\beta \rightarrow 0$  afterwards [277].



FIGURE 34: Diffraction of a monochromatic wave. A wave of wavelength  $\lambda$  travelling in the positive z-direction is diffracted by a pupil of size  $\emptyset = \lambda$ . Insets: intensity cuts at the indicated distances (linear scale). Calculations using angular spectrum propagation and Fresnel diffraction are visually indistinguishable on this scale. In the far-field (*right*), the Fraunhofer pattern is visible.

which is known as the Fraunhofer integral. We see that propagating the field of an aperture to a far distant screen corresponds to performing an optical Fourier transform, where angular frequencies are mapped to position and vice versa.

## B.3 GENERALISED IMAGE FORMATION

Now we can try to build a general model of how the (real) image formation procedes. In the Huygens picture, an imaging system converts a spherical wavelet emanating from a point in the object plane into a spherical wavelet converging towards a point on the image plane. Ideally, there is a purely linear relation between the distance of two points (P, Q) in the object plane and the respective points (P', Q') in the image plane,  $\overline{P'Q'} = |M|\overline{PQ}$ , where *M* is called the magnification of the system and for M < 0 the image appears inverted.

The simplest device that achieves such a mapping is a spherical lens. Its action is to imprint a space-dependent phase delay onto an incoming wave front: where the lens has a thickness D, the optical path length d appears prolonged,  $d = n_{\rm r}D$ , by the index of refraction  $n_{\rm r}$ . In the paraxial approximation, i.e. where it is valid to approximate the spherical surfaces of radii  $R_1, R_2$  lens by parabolic surfaces, the phase imparted on a wavefront becomes [112, § 5.1]

$$t_{\ell}(x,y) = e^{-\frac{ik}{2f}(x^2+y^2)}, \text{ where } \frac{1}{f} = (n_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
 (188)

is the focal length. By this, and using again the pupil function (175), the field U(x, y) immediately in front of the lens gets transformed into a field

$$U_{\Sigma}(x,y) = t_{\ell}(x,y)P(x,y)U(x,y)$$
(189)

Lensmaker formula immediately behind the lens. If we use the Fresnel integral (Eq. 179) to propagate this field to the back focal plane (z = f), we obtain

$$U(x,y,f) = \frac{1}{i\lambda f} e^{\frac{ik}{2f}(x^2 + y^2)} \iint_{-\infty}^{+\infty} U_{\Sigma}(x',y') e^{-\frac{ik}{f}(xx' + yy')} dx' dy'$$
(190)

$$=\frac{\mathrm{e}^{\mathrm{i}\kappa z}}{\mathrm{i}\lambda z}\mathrm{e}^{\frac{\mathrm{i}\kappa}{2z}(x^{2}+y^{2})}\mathscr{F}\left\{U_{\Sigma}(x',y')\right\}_{\frac{x}{\lambda f},\frac{y}{\lambda f}}$$
(191)

which is exactly the Fraunhofer diffraction (Eq. 186). This means that the field in the focus plane of a lens is the same as the (scaled) field without lens at large distance.



FIGURE 35: Irradiance profile through the focus of a lens. Fresnel-integrated field of a homogeneous light field of wavelength  $\lambda$ , for a lens with diameter  $\emptyset$  =  $2.5 \times 10^3 \lambda$  and focal length  $f = 5 \times$  $10^4\lambda$ .

Every sort of imaging system must have a phase-retarding property similar to Eq. 188, which turns diverging into converging light cones. In the framework of linear-system theory, similar to the space-domain field-propagation picture (Eq. 171), one can make the general ansatz that the image amplitude is represented by a superposition integral [112, 193]

Convolution property of imaging systems

$$U_{i}(x,y) = \iint_{-\infty}^{+\infty} h(x,y;x',y') U_{o}(x',y') dx' dy'$$
(192)

where the impulse response is for now an arbitrary function of coordinates in the object (x', y')and the image plane (x, y). We know that corresponding distances in object and image plane are connected via the magnification *M*, hence it is useful to introduce reduced coordinates  $(\xi, \eta)$  in the object plane [112]. With these, the scaled image  $U_g$  predicted by geometrical (ray) optics<sup>3</sup> reads

$$\xi = Mx', \quad \eta = My' \implies U_{g}(\xi, \eta) = \frac{1}{|M|} U_{o}(\xi/M, \eta/M).$$
(193)

A rigorous treatment [112] shows that in planes connected via the lens law of geometrical optics<sup>4</sup> and using approximations that are usually well justified for realistic imaging settings, the impulse response takes the form

$$h(x,y;x',y') \longrightarrow h(x-\xi,y-\eta) \tag{194}$$

3 I.e., disregarding diffraction.

<sup>4</sup>  $\frac{1}{z_1} + \frac{1}{z_2} - \frac{1}{f} = 0$  for planes at  $z_1, z_2$  and focal length f.

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making the superposition integral (192) look like a convolution:

$$U_{i}(x,y) = \iint_{-\infty}^{+\infty} h(x-\xi,y-\eta) U_{o}(\xi,\eta) d\xi d\eta$$

$$= U_{g}(x,y) * h(x,y)$$
(195)
(196)

This form has an important intuitive interpretation: The image produced by an (aberration-free) imaging system is inverted and scaled by the magnification; the effects of diffraction are included by convolving the ray-optics image with the impulse response of the imaging system.

## B.3.1 Coherent and incoherent imaging

If we want to record an image, we need an image detector. The exposure time to accumulate sufficient signal is typically many orders of magnitude larger than the inverse bandwidth<sup>5</sup>, which characterises how fast two different frequency components present 'run out of phase'. This means, the recorded intensity is the time-averaged instantaneous intensity,

$$I_{i}(x,y) = \langle |U_{g}(x,y;t)\rangle_{t}.$$
(197)

How this time average needs to be calculated, depends on the illumination. When illumination is coherent, the superposing wavelets oscillate in unison, and therefore must be added on a complex-amplitude level [112]. A coherent imaging system is therefore linear in complex amplitude:

$$I_{i}(x,y) = |U_{g}(x,y) * h(x,y)|^{2}$$
(198)

If illumination is incoherent, on the other hand, the individual wavelets oscillate in an uncorrelated manner, and must be added on an intensity level. Therefore, an incoherent imaging system is linear in intensity, and the intensity impulse response is the modulus-squared of the complexamplitude impulse response:

$$I_{i}(x,y) = \left| U_{g}(x,y) \right|^{2} * \left| h(x,y) \right|^{2} = I_{g}(x,y) * \left| h(x,y) \right|^{2}$$
(199)

The intensity impulse response is in applied optics termed the *point spread function* (PSF) of the imaging system due to its intuitive interpretation. The PSF can be measured in the laboratory to determine the quality of an imaging system (see  $\S$ § 5.2.5, 11.3)

### B.3.2 Resolution limit of an ideal system

An important consequence in the context of imaging is that the width of any structure in the image is limited by the number of Fourier component contributing to it. For example, applying the Fraunhofer integral<sup>6</sup> for a circular aperture of radius R, illuminated homogeneously from the back, yields for the field and intensity on a screen in distance z

$$U(\rho) = \frac{\pi R^2}{i\lambda z} e^{ikz} e^{ik\rho^2/2z} \frac{2J_1(k\rho R/z)}{k\rho R/z} \quad \text{and}$$
(200)

$$I(\rho) = \left(\frac{\pi R^2}{\lambda z}\right)^2 \left(\frac{2J_1(k\rho R/z)}{k\rho R/z}\right)^2,$$
(201)

<sup>5</sup> For single-mode lasers, this corresponds to the line width.

<sup>6</sup> In this case of cylindrical symmetry, the Fourier transform is conveniently replaced by a Fourier–Bessel transform [112], which for a circular disc directly gives the solution (200).

where  $J_1$  is the first-order Bessel function of the first kind and  $\rho$  is the radial coordinate in the image plane. This intensity distribution (see Fig. 36) is called an Airy pattern.



FIGURE 36: *Airy pattern*. Relative intensity plot of the innermost Airy rings in (a) linear and (b) logarithmic scale. A cut through the maximum is shown in (c), where the intensity is plotted against the dimensionless coordinate  $x \equiv k\rho R/z$  and R/z corresponds to an NA of 0.5. The zeros are located where *x* is a multiple of  $\approx$  3.8317.

As we have seen in § B.3, the image-plane field is given by the magnified object-plane field convolved with the impulse response. For a diffraction-limited system, where aberrations are negligible, the intensity impulse response (or PSF) is directly given through the Airy pattern. The best achievable resolution depends on the size of the Airy pattern, i. e., the size of the exit pupil determined by the numerical aperture (NA).

An indication when two point sources can be resolved by an imaging system of circular aperture is given by the Rayleigh criterion. The Rayleigh criterion presumes that the smallest distance between two Airy patterns that still allows to distinguished them is reached when the maximum of the one Airy pattern falls on the first minimum of the other [269]. In the general, non-paraxial case, this distance, also know as the Rayleigh radius, accounts to

$$d_0 = 0.61 \frac{\lambda}{\text{NA}},\tag{202}$$

where NA is the limiting numerical aperture of the imaging system [112].

### в.3.3 *Effect of aberrations*

As seen in §B.3, the ideal (aberration-free) image is obtained by propagating the field of the exit pupil into the image plane using the Fraunhofer formula (186). It turns out that even if some aberrations are present *somewhere* in the imaging path, it is sufficient to treat the exit pupil as if it were illuminated by a perfect spherical and imagining a phase-shifting plate at the pupil position which imprints all the aberrations onto the through-passing field. This phase-shifted field can then be propagated to the image plane using the standard Fraunhofer integral [112, 193]. The corresponding generalised pupil function for such a case reads

$$\mathscr{P}(\mathbf{x}, \mathbf{y}) = P(\mathbf{x}, \mathbf{y}) \mathbf{e}^{ikW(\mathbf{x}, \mathbf{y})},\tag{203}$$

with the aberration function W. W can be understood as the phase difference of the aberrated field at a point (x, y) in the pupil compared to the ideal field without aberration, the so-called Gaussian reference sphere [112].

For cylindrically symmetric systems, it is convenient to express *W* in normalised polar coordinates  $(0 \le \rho \le 1 \text{ and } 0 \le \varphi < 2\pi)$ . An example for a popular basis is given by the Zernike circle polynomials  $Z_i$  [42, 193, 302],

$$W(\rho,\varphi) = \sum_{i=1}^{\infty} c_i Z_i(\rho,\varphi), \tag{204}$$

here noted using Noll's sequential indexing [206]. The Zernike polynomials form an orthonormal basis on the unit disc,

$$\frac{1}{\pi} \int_0^{2\pi} \int_0^1 Z_i(\rho, \varphi) Z_k(\rho, \varphi) d\rho d\varphi = \delta_{ij},$$
(205)

which makes it straight forward to decompose a measured aberration function W into its Zernike components,

$$c_i = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 W(\rho, \varphi) Z_i(\rho, \varphi) d\rho d\varphi.$$
(206)

This is useful for quantification of the aberrations present in an optical system as, e.g., carried out in Refs [193, 247].



FIGURE 37: The first 10 Zernike polynomials. Top-down: Increasing radial degree. Left-right: Increasing azimuthal degree. Colour scale varying from -1 (blue) to +1 (red).

### **B.4 IMAGING SYSTEM IMPLEMENTATION**

The alignment procedure for the vertical main-chamber imaging system ( $\S$  5.2.5) is given below. Here, *z* is understood as the respective *local* beam propagation direction.

- 1. Use a tight probe beam,<sup>7</sup> align it onto the crossed ODT by minimising the time needed to blow the atoms away with resonant light.
- 2. Walk the probe beam to till normal incidence on vacuum window by looking at back-reflexions; iterate with step 1 until content.
- 3. Insert objective, bring close to nominal *z*-position and zero tilt by hand.

<sup>7</sup> We are currently using a TC06APC-405 triplet fibre-optic collimator from Thorlabs, Inc., with 1.1 mm waist diameter  $(1/e^2)$ .

- 4. Adjust objective *xy*-position by bringing probe beam transmission on optical axis ('centre of tube').
- 5. Look at back-reflexions of all lens surfaces from within objective and try to collapse them onto each other by adjusting tilt axes; iterate with step 4 until content.
- 6. Insert imaging lens, iteratively align *xy*-direction by looking at transmission, tilt by looking at back-reflexion.
- 7. Position camera as well as possible at focal distance from imaging lens, steer beam onto sensor using mirrors.
- 8. Fine-adjust *z*-position of objective using stepper motor until sharp in-situ image of trapped atoms is attained.

# ADDITIONAL PUBLICATIONS

C.1 DIPOLAR QUANTUM MIXTURES OF ERBIUM AND DYSPROSIUM ATOMS

The following manuscript has appeared in

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C



(a) <sup>166</sup>Er (d) (b) (c) ĩ (e) <sup>164</sup>Dy 164 Dy (h)<sup>166</sup>Er - <sup>164</sup>Dy 0.8 (f) 166Er - 164Dy (g)<sup>166</sup>Fr 0.6 04 0.2

FIG. 1. Binary Bose-Einstein condensation in a <sup>166</sup>Er-<sup>164</sup>Dy mixture. (a)-(c) Pairs of TOF absorption images at different evaporation stages, showing (a) a thermal mixture at about 180 nK, (b) an Er cloud at the onset of condensation coexisting with a thermal Dy gas at about 80 nK, and (c) the binary dipolar BEC with total atom numbers  $N = 3.4 \times 10^4 (2.6 \times 10^4)$  for Er(Dv) with condensate fractions of about 45%,  $\tilde{x}$  denotes the horizontal axis perpendicular to the imaging axis. (d),(e) Density profiles integrated along z, extracted from (c). Solid lines depict the 1D bimodal fit, the dotted lines show Gaussian fits to the thermal components. (f)-(h) BECs with a controlled number imbalance giving about  $N^{\text{Er}} = (3.2, 6.4, 9.2) \times 10^4$  with (35, 70, 85)% condensate fraction and  $N^{\text{Dy}} = (3.1, 2.9, 0.9) \times$ 10<sup>4</sup> with (30, 55, 30)% condensate fraction for (f)-(h), respectively. The deformations and the relative displacement of the clouds are caused by interspecies interaction (see main text). The color bar indicates the optical density.

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range contact interactions are at play.

intriguing physics within reach comes with comparatively simple experimental approaches to achieve quantum degen-

eracy. Several research groups have either recently reported

on new experimental realizations of quantum gases with Dy [10,11] or Er [12] or are actively pursuing it [13,14]. An alternative route to access dipolar quantum physics is

provided by polar molecules, possessing an electric dipole moment. Up to now, ultracold gases of polar molecules

have been created from nondipolar binary quantum mix-

tures of alkali atoms [15-18] and dipolar spin-exchange

interactions have been recently observed with lattice-

confined molecules [19]. Besides molecule creation, heteronuclear quantum mixtures have been used as powerful

resources to realize a broad class of many-body quantum

states (e.g., [20-27]), in which intra- and interspecies short-

the first time the physics of heteronuclear mixtures with

the one of magnetic dipolar quantum gases. Our motivations to create quantum mixtures by combining two differ-

ent dipolar species, Er and Dy, are numerous. First, the

coupling between the two components acquires an aniso-

tropic and long-range character due to the strong inter-

species DDI, in contrast to purely contact-interacting

mixtures. The emergent physical richness of the system

has only begun to be uncovered by theory. Recent studies

include the prediction of anisotropic boundaries in the

dipolar immiscibility-miscibility phase diagram [28,29],

In the experiment described in this Letter, we merge for

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that, in mixture experiments, the imbalance in dipolar strength, combined with the interspecies interactions, promises fascinating prospects for creating long-lived quantum-droplet states and for accessing exotic fermionic superfluidity, for which the degree of deformation of the Fermi surface is species-dependent [5].

Second, the rich, but different atomic spectra of Er and Dy open promising prospects for implementing speciesdependent optical manipulations schemes, including species-selective optical potentials at magic wavelengths and checkerboard-pattern-like lattice structures. Third, Er and Dy feature many stable bosonic and fermionic isotopes (both elements have four isotopes with natural abundances above 15%). Such an isotope variety allows us to create dipolar Bose-Bose, Bose-Fermi, and Fermi-Fermi heteronuclear mixtures. Last, mixtures composed of two different magnetic species serve as an ideal platform to produce ground-state polar molecules with both an electric and magnetic dipole moment, offering novel degrees of control and competing long-range interactions [35–37].

We here report on the first experimental realization of quantum-degenerate dipolar mixtures of Er and Dy atoms, using an all-optical approach for trapping and cooling. Taking advantage of the isotope richness in Er and Dy, we produce dipolar Bose-Bose mixtures with five different isotope combinations, as well as one Bose-Fermi mixture. We note that, prior to this work, the production of a Dy isotope mixture of a degenerate Fermi gas and a Bose gas near condensation has been reported [2] and, more recently, a doubly degenerate Fermi-Fermi mixture has been created from two Er spin states [39]. Experimental efforts are also devoted to creating Dy-K mixtures [40].

In the following, we detail the production of a double dipolar Bose-Einstein condensate (ddBEC) of <sup>166</sup>Er and <sup>164</sup>Dy. The same procedure is used for the other isotope mixtures. Our experiment starts with a double magneto-optical trap (MOT) of Er and Dy, as reported in our recent work [41]. For both species, the MOT operates on narrow intercombination lines and yields cold and spin-polarized samples in the absolute lowest Zeeman sublevels [10,13,41,42]. After loading the double MOT, we optically compress the mixture in 400 ms (cMOT phase) by reducing the detuning and power of the MOT beams as well as the magnetic-field gradient.

We then transfer the mixture into an optical dipole trap (ODT) by superposing it with the cMOT for 100 ms. Initially, the ODT consists of a single laser beam at 1064 nm, propagating along the horizontal (*y*) axis. The beam has a fixed vertical (*z*) focus of about 22  $\mu$ m, whereas the horizontal waist can be controlled via a time-averaging-potential technique (see, e.g., [43]). This leads to an elliptic beam with variable aspect ratio (AR). Best transfer efficiency is observed for a beam power of 32 W and an AR of 4, which provides good spatial overlap between the cMOT and the ODT. We then switch off the MOT beams and magnetic-field

gradient, and start a 5-sec evaporation sequence, during which we apply a bias magnetic field  $B_{\text{evap}}$  along the gravity (*z*) axis to preserve spin polarization.

Our strategy for evaporative cooling can be divided into three main stages. (i) During the initial 600 ms, we reduce the AR to unity while lowering the power of the singlebeam ODT. This increases the density of the mixture at a roughly constant trap depth. (ii) We start forced evaporation in the horizontal ODT and add a vertically propagating dipole trap beam. The vertical beam is derived from the same laser source as the horizontal one and has a power of 15 W and a waist of 130  $\mu$ m. (iii) We proceed with forced evaporation in the crossed ODT by reducing the powers of both beams nearly exponentially until the mixture is close to quantum degeneracy. In the final stage of the evaporation, we increase the AR to 5 to create a pancakelike trapping geometry and further decrease the trap depth until we reach double quantum degeneracy. To probe the atomic mixture, we switch off the ODT and, after a time-of-flight (TOF) expansion of 25 ms, we perform sequential absorption imaging with a resonant light pulse at 401 nm for Er and 500  $\mu$ s later at 421 nm for Dy [3,44]; both pulses have a duration of 50  $\mu$ s. The imaging light propagates horizontally with an angle of  $45^{\circ}$  with respect to the y axis.

Unlike many alkali mixtures [45-48], Er and Dy exhibit very comparable atomic polarizabilities  $\alpha$  because of their similar atomic spectra. From single-species experiments [14,49], a ratio  $\alpha_{\rm Dy}/\alpha_{\rm Er} = 1.06$  at 1064 nm is expected. For our initial ODT parameters, we calculate trap frequencies of about  $\nu^{\text{Er}} = (490, 5, 1980)$  Hz and  $\nu^{\text{Dy}} = (505, 5, 5)$ 2050) Hz [50], corresponding to trap depths of 380 and 410  $\mu$ K for Er and Dy, respectively. Although small, the difference in trap depths has an important effect on the evaporation trajectory of the mixture. We observe that the more weakly trapped Er atoms act as a coolant for Dy and are preferentially evaporated from the trap ("sympathetic losses" [47,51]). To sustain Er atom numbers high enough to achieve double quantum degeneracy, we imbalance the initial atom number in the MOT with Er as the majority component. The atom number imbalance can be easily controlled by individually changing the MOT loading time and beam power. This strategy is often employed in multispecies experiments, e.g., [52,53].

Figures 1(a)–1(c) show the phase transition from a thermal Er-Dy mixture (a) to a ddBEC (c). The TOF absorption images reveal the textbooklike fingerprint of condensation, the emergence of a bimodal density distribution, as plotted in Fig. 1(d). The condensation series [Figs. 1(a)–1(c)] is taken for an Er(Dy) MOT loading time of 3 s (1 s), for which we transfer  $8 \times 10^6$  (7 × 10<sup>5</sup>) Er(Dy) atoms into our ODT and measure a temperature of about  $35 \ \mu$ K; this parameter set allows us to create numberbalanced ddBECs. In agreement with the expected polarizabilities, we measure ODT trap frequencies of  $\nu^{\rm Er} = (48.6(3), 29.7(9), 144(1))$  Hz and  $\nu^{\rm Dy} = (50.6(3), 30.2(9),$ 

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160(1)) Hz [54]. The resultant gravitational sag between the two species is 2.1(2)  $\mu$ m. By varying the imbalance of the MOT loading, we can produce degenerate mixtures with different atom number ratios and condensate fractions, which is exemplified in Figs. 1(f)–1(h). For large condensates, one directly observes a deformation of the density profiles due to interspecies interaction, as we discuss later in more detail.

To quantify the cooling efficiency, we plot the normalized phase-space density (PSD/PSD<sub>0</sub>) as a function of normalized atom numbers  $(N/N_0)$  during the evaporation stages ii and iii [see Fig. 2(a)].  $PSD_0$  and  $N_0$  are the respective initial values at stage ii. From this plot, we extract  $\gamma = -d \ln(\text{PSD}/\text{PSD}_0)/d \ln(N/N_0)$  [55], which captures the evaporation efficiency, via a linear fit to the data. In the single-beam ODT (stage ii), we see similar efficiencies both in mixture and single-species operations, with  $\gamma \approx 1.2$ . In the crossed ODT (stage iii), we find  $\gamma^{\text{Er}} =$ 2.4(9) for Er in the mixture operation. This value is comparable to state-of-the-art single-species Er experiments [56] and, as expected, little affected by a small admixture of Dy atoms. Contrarily, the cooling efficiency of Dy in stage (iii) strongly benefits from the sympathetic cooling by Er: We observe a steep increase of the Dy PSD in the mixture and extract  $\gamma_{sym}^{Dy} = 7(2)$ , whereas for the



FIG. 2. (a) Evaporation trajectories: PSD/PSD<sub>0</sub> as a function of  $N/N_0$ . Filled squares (circles) indicate the Er(Dy) trajectory in mixture operation. The lines are linear fits to the data for evaporation in the single-beam (ii) and crossed-beam (iii) ODT (see main text). Open symbols show the single-species operation for Er (squares) and for Dy with small (circles) and large (diamonds) initial atom numbers. In the latter case, Dy condenses alone. (b) Atom numbers in the mixture of Er (red) and Dy (blue) at the onset of condensation as a function of the magnetic-field value during evaporation. Condensation is reached for atom numbers above about  $3.5 \times 10^4$  (gray region). We record the best performance for a ddBEC around 2.075 G. Arrows indicate the position of known single-species Feshbach resonances [57–59].

same  $N_0^{\text{Dy}}$  but in single-species operation, the evaporation efficiency is considerably lower and would not suffice for condensation. However, with higher  $N_0^{\text{Dy}}$  we can still produce large Dy BECs in single-species operation.

The proper choice of  $B_{evap}$  plays an important role for cooling magnetic rare-earth atoms and becomes even more critical in mixture operation. It has indeed been observed in single-species experiments [57-59] that both Er and Dy exhibit extremely dense and temperature-dependent spectra of homonuclear Feshbach resonances. Figure 2(b) shows the atom numbers of the 166Er-164Dy mixture at the onset of condensation as a function of  $B_{evap}$  in a small magneticfield range from 0.5 to 5 G. As expected, we find a number of broad and narrow loss features. Some of them are connected to known homonuclear Feshbach resonances [57–59], others we attribute to unknown high-temperature resonances or detrimental interspecies scattering conditions. In a few narrow magnetic-field windows, we observe atom numbers large enough for both components to condense. Our magnetic-field stability of about 2 mG is sufficient to reliably operate in most of these small windows. The optimal value of  $B_{evap}$ , listed in Table I, depends on the isotope combination.

Combining Er and Dy offers an unprecedented variety of heteronuclear mixtures with 16 possible isotope configurations, including Bose-Bose, Bose-Fermi, and Fermi-Fermi quantum gases (see Table I). Using the cooling and trapping procedure optimized for <sup>166</sup>Er-<sup>164</sup>Dy, we are able to produce five ddBECs and one Bose-Fermi mixture. Concerning the remaining combinations, we know from previous experiments that both <sup>167</sup>Er and <sup>163</sup>Dy need a different experimental approach since <sup>167</sup>Er undergoes light-induced losses in a 1064-nm ODT [4], whereas an inverted hyperfine structure, requiring most probably additional optical pumping stages. Both isotopes will be investigated for future studies of Fermi-Fermi mixtures.

Figures 3(a)-3(d) show absorption pictures of our doubly degenerate isotope mixtures. We are able to

TABLE I. (Left) List of optimal  $B_{evap}$  and  $\gamma_{sym}^{Dy}$  for the quantumdegenerate Er-Dy mixtures. (Right) Chart of the available isotope mixtures: ( $\checkmark$ ) realized double-degenerate mixtures, ( $\times$ ) thermal mixtures, where degeneracy is not yet reached. (...) Mixtures with <sup>167</sup>Er and <sup>163</sup>Dy are not investigated here.

| mixture                              | $B_{\rm evap}({\rm G})$ | $\gamma_{\rm sym}^{\rm Dy}$ | Dy\Er | 166          | 167 | 168          | 170          |
|--------------------------------------|-------------------------|-----------------------------|-------|--------------|-----|--------------|--------------|
| <sup>166</sup> Er- <sup>164</sup> Dy | 2.075                   | 7(2)                        | 161   | ×            |     | $\checkmark$ | ×            |
| <sup>168</sup> Er- <sup>164</sup> Dy | $3.300 \\ 3.300$        | $6(2) \\ 6(2)$              | 162   | ×            |     | $\checkmark$ | $\checkmark$ |
| <sup>170</sup> Er- <sup>162</sup> Dy | 1.540                   | 11(7)                       | 163   |              |     |              |              |
| $^{168}\text{Er-}^{161}\text{Dy}$    | $3.210 \\ 3.455$        | 4(1)                        | 164   | $\checkmark$ |     | $\checkmark$ | $\checkmark$ |

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FIG. 3. Absorption pictures of the double-degenerate Bose-Bose mixtures (a)–(d) and the Bose-Fermi mixture (e). The pictures are averaged over 5–10 single shots. For all combinations, degeneracy is reached with the evaporation ramp optimized for the <sup>166</sup>Er-<sup>164</sup>Dy mixture (cf. Fig. 1).  $B_{evap}$  is listed in Table I. Typical condensate fractions are around 30%, total atom numbers range between  $1 \times 10^4$  and  $3.5 \times 10^4$  atoms. For the imbalanced case, higher condensate fractions can be achieved (see Fig. 1). For the <sup>161</sup>Dy Fermi gas,  $N = 8 \times 10^3$ ,  $T/T_F \approx 0.5$ , and TOF = 15 ms.

condense all Bose-Bose isotope mixtures with the exception of <sup>166</sup>Er-<sup>162</sup>Dy, for which we record severe losses during the evaporation, potentially due to a very large interspecies scattering length. For all degenerate mixtures, we observe sympathetic cooling of Dy by Er. The atom numbers in the ddBECs differ significantly for the different mixtures, while the initial atom numbers in the MOT are very similar. This points to different intra- and interspecies scattering properties during evaporation. The optimal  $B_{\rm evap}$  and the extracted  $\gamma_{\rm sym}^{\rm Dy}$  are listed in Table I.

We also prepare one Bose-Fermi mixture [see Fig. 3(e)], in which a <sup>168</sup>Er BEC coexists with a degenerate Fermi gas of <sup>161</sup>Dy. Although the cooling process of spin-polarized fermions can differ substantially from bosons, we are able to reach Bose-Fermi degeneracy with a similar evaporation scheme [60]. We measure a temperature of the Fermi gas of  $T/T_F \approx 0.5$ , with the Fermi temperature  $T_F = 140$  nK. We expect that deeper degeneracy might be reached by using smaller ODT beam waists [4].

Remarkably, in the TOF images in Figs. 1 and 3 hints of interspecies interactions can be spotted: in mixture operation, the center-of-mass (c.m.) position of each BEC is vertically displaced with respect to its thermal-cloud center [see also Fig. 4(a)]. The two BECs are displaced in opposite directions, with the heavier (lighter) Er(Dy) always shifted down (up). Contrarily, in single-species operation the condensates and their thermal clouds are centered [see Figs. 4(b) and 4(c)].

To confirm that the displacement after TOF originates from in-trap interspecies interaction, we prepare a ddBEC, let it equilibrate for 50 ms, and then selectively remove either of the two species from the ODT using a resonant light pulse [61]. After a variable hold time in the ODT, we release the remaining cloud and record its c.m. position after TOF. As shown in Figs. 4(d) and 4(e), we observe a



FIG. 4. Evidence of interspecies interactions in the  $^{166}\text{Er}$ - $^{164}\text{Dy}$  mixture: absorption pictures of Er and Dy in mixture (a) and single-species (b),(c) operation. (d),(e) Filled symbols show the c.m. position along z of the Dy BEC (d) and the Er BEC (e) after removal of the other species with resonant light. The gray region indicates the transient time until full removal. The solid lines are damped sine fits to the oscillations. For comparison, open symbols show the c.m. position in a thermal mixture.

very pronounced c.m. oscillation of the remaining BEC component with a frequency close to its bare trap frequency. The oscillations of Er [removing Dy, Fig. 4(d)] and of Dy [removing Er, Fig. 4(e)] proceed in counterphase, as expected from their initial separation in trap. Repeating the same measurement with a thermal-thermal mixture, or a mixture with just one condensed component (not shown), yields negligible or significantly weaker oscillations, respectively.

The spatial separation between the two condensed components and their oscillating behavior after removal indicate that, for our trap geometry, the overall interspecies interaction-contact plus dipolar-has a repulsive character. We note that the interspecies scattering length, governing the contact interaction, and its Feshbach tuning are presently unknown and beyond reach of state-of-the-art scattering models [57,62]. To isolate the different sources of interaction and determine their signs, future dedicated experiments studying the interplay between trap geometry, dipole orientation, and interspecies scattering length, combined with simulations based on generalized coupled Gross-Pitaeskvii equations are necessary. Indeed, the DDI breaks the angular symmetry of the mean-field interspecies potentials and is expected to render the strength and the sign of the overall interspecies interaction anisotropic and trap dependent.

In conclusion, we have produced heteronuclear dipolar quantum mixtures by combining two strongly magnetic atomic species, Er and Dy. Their isotope variety, the richness of their interactions, the imbalance in the dipolar strength, and simple laser-cooling schemes make Er-Dy mixtures a powerful experimental platform to access manybody quantum phenomena, in which contact and dipolar intra- and interspecies interactions are at play.

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## XXXII ADDITIONAL PUBLICATIONS

## C.2 FESHBACH RESONANCES IN AN ERBIUM-DYSPROSIUM DIPOLAR MIXTURE

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#### PHYSICAL REVIEW A 102, 033330 (2020)

### Feshbach resonances in an erbium-dysprosium dipolar mixture

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We report on the observation of heteronuclear magnetic Feshbach resonances in several isotope mixtures of the highly magnetic elements erbium and dysprosium. Among many narrow features, we identify two resonances with a width greater than one Gauss. We characterize one of these resonances, in a mixture of <sup>168</sup>Er and <sup>164</sup>Dy, in terms of loss rates and elastic cross section, and observe a temperature dependence of the on-resonance loss rate suggestive of a universal scaling associated with broad resonances. Our observations hold promise for the use of such a resonance for tuning the interspecies scattering properties in a dipolar mixture. We further compare the prevalence of narrow resonances in an <sup>166</sup>Er - <sup>164</sup>Dy mixture to the single-species case, and observe an increased density of resonances in the mixture.

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### I. INTRODUCTION

Ultracold quantum gases are a highly successful platform for physics research largely because it is possible to create simplified and controllable versions of condensed matter systems [1]. As the field has advanced, great progress has been made by reintroducing complexity in a carefully controlled manner. This complexity can manifest in the form of interparticle interactions [2–4], the species and statistics of the particle under study [5–7], or in the form of the potential landscape, control protocols, and imaging techniques applied to the system [8,9]. In this work, we explore interspecies Feshbach resonances as a means of generating tunable interactions between two different species of complex dipolar atoms.

Atoms with large magnetic dipole moments, such as the lanthanide series elements erbium and dysprosium, interact in a manner that is both long-range and anisotropic. This is in contrast to more commonly used atomic species, such as alkali and alkaline earth metals, which primarily interact in a short-range and isotropic way. The recent creation of degenerate Bose and Fermi gases of such atoms [10–13] has enabled the observation of a wealth of phenomena including quantum-stabilized droplet states [14–16], roton quasiparticles [17], supersolid states [18–20], and a nonisotropic Fermi surface [21].

In a separate direction, degenerate mixtures of multiple atomic species have also provided diverse opportunities for the study of physical phenomena. Examples include studies of polarons that arise when an impurity species interacts with a background gas [22–27], and the formation of heteronuclear molecules with large electric dipole moments [28–31].

We expect that combining dipolar interactions with heteronuclear mixtures will lead to a rich set of physical

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phenomena, the exploration of which has only recently begun. In particular, dipolar interactions are expected to have dramatic consequences for the miscibility of binary condensates [32–34], and in turn on vortex lattices that arise in such systems [35]. Further, certain properties of polarons are predicted to emerge when either the background [36] or both background and impurity [37] particles experience dipolar interactions [38].

Dipolar heteronuclear mixtures have recently been demonstrated [39], but so far the interspecies scattering properties have not been explored, either experimentally or theoretically. In these complex dipolar species, scattering properties are dictated by both anisotropic long-range dipolar interactions, which can be tuned through a combination of system geometry and magnetic field angle, and by contact interactions, which can be tuned through the use of interspecies Feshbach resonances. While scattering models and experimental demonstrations exist for mixtures of single- and two-valence electron atoms (which lack strong dipolar interactions) [40,41], the scenario of two multivalence electron atoms has yet to be considered, and represents a frontier for our understanding of ultracold scattering. In many commonly used atomic systems, the strength, character, and location of magnetic Feshbach resonances can be predicted with high precision through coupled-channel calculations [3]. However, the complexity of the internal level structure and coupling mechanisms present in lanthanide atoms lead to significant challenges for the development of a microscopic theory with predictive power, and so necessitate an experimental survey to find resonances with favorable properties [42-46].

To this end, we searched for heteronuclear Feshbach resonances broad enough to provide a practical means for tuning the interspecies interaction in Bose-Bose and Bose-Fermi dipolar quantum mixtures. Using atomic-loss spectroscopy to identify resonances, we perform surveys of fermionic <sup>161</sup>Dy and bosonic <sup>164</sup>Dy together with <sup>166</sup>Er, <sup>168</sup>Er, and <sup>170</sup>Er over

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TABLE I. Comparatively broad resonances found in specific isotope mixtures together with estimated center positions and widths (FWHM) from Gaussian fits to atom loss spectra. Each value is an average between the fit values of Er and Dy.

| Combination                           | Resonance magnetic field (G) | Width (G) |  |  |
|---------------------------------------|------------------------------|-----------|--|--|
| <sup>168</sup> Er - <sup>164</sup> Dy | 13.32(4)                     | 1.7(1)    |  |  |
| <sup>166</sup> Er - <sup>164</sup> Dy | 34.09(3)                     | 1.5(1)    |  |  |
| <sup>166</sup> Er - <sup>161</sup> Dy | 161.31(3)                    | 0.84(9)   |  |  |
| <sup>168</sup> Er - <sup>161</sup> Dy | 161.30(2)                    | 0.93(5)   |  |  |
| <sup>170</sup> Er - <sup>161</sup> Dy | 161.26(3)                    | 0.91(8)   |  |  |

a magnetic-field range from zero to several hundred gauss (the exact range varies by isotope combination due to availability of favorable evaporation conditions). We also explored a Fermi-Fermi mixture of <sup>167</sup>Er and <sup>161</sup>Dy, but observed no broad resonances there. In Table I we summarize positions and widths of these features observed in our surveys. As an exemplary case, we present a more detailed characterization of the resonance near 13.5 G in the <sup>168</sup>Er - <sup>164</sup>Dy Bose-Bose mixture, through measurements of interspecies thermalization and the dependence of atomic loss on temperature.

In addition, our dipolar mixtures host a large number of narrow interspecies resonances. In previous experiments with single species, the density and spacing of these narrow resonances has been studied to reveal a pseudorandom distribution that can be modeled well using random matrices [43,45,46]. By performing high resolution scans over specific magneticfield ranges, we find that the average density of interspecies resonances exceeds the combined density of intraspecies resonances, perhaps indicating the contribution of odd partial waves or molecular states with antisymmetric electron configurations for the interspecies case, which are not present in the scattering of identical bosons.

Finally, in each Fermi-Bose mixture involving <sup>161</sup>Dy we observe a correlated loss feature between fermionic Dy and bosonic Er atoms. Strangely, the loss feature is present at the same magnetic-field value for all three bosonic erbium isotopes studied. Such behavior is inconsistent with a typical interspecies Feshbach resonance, where the magnetic field at which the resonance occurs is strongly dependent on the reduced mass of the atoms involved [47]. The mechanism behind this unusual feature is as of yet unknown and calls for further experimental and theoretical investigations.

#### II. OBSERVATION OF INTERSPECIES RESONANCES

Our experimental sequence is similar to the one introduced in our previous works [39,48]. After cooling the desired isotope combination of erbium and dysprosium atoms in a dual-species magneto-optical trap (MOT), we load the atoms into a crossed optical dipole trap (ODT) created by 1064-nm laser light. Here we perform evaporative cooling down to the desired sample temperature. During the whole evaporation sequence, we apply a constant and homogeneous magnetic field ( $B_{ev}$ ), pointing along the z direction opposite to gravity.  $B_{ev}$ preserves the spin polarization in the lowest Zeeman sublevel

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of both species. We use different values of  $B_{\rm ev}$  to optimize the evaporation efficiency depending on the isotope combination and on the range of the target magnetic field  $(B_{\rm FB})$  to be investigated. The final ODT has trap frequencies  $\omega_{x,y,z} =$  $2\pi \times (222, 24, 194)s^{-1}$ . We typically obtain mixtures with atom numbers ranging from  $3 \times 10^4$  to  $1 \times 10^5$  atoms for each species. The sample is in thermal equilibrium at about 500 nK, which corresponds to about twice the critical temperature for condensation. Typical densities are up to a few  $\times 10^{12}$  cm<sup>-3</sup> for each species. After preparing the mixture, we linearly ramp the magnetic field from  $B_{ev}$  to  $B_{FB}$  in 5 ms, either in an increasing or decreasing manner. The current flowing in the coils that generate the magnetic fields can be changed on the millisecond time scale, and the field at the position of the atoms settles to the part-per-thousand level in approximately 10 ms. We hold the mixture for a time ranging between 5 ms and 400 ms depending on the experiment. At the end of the hold time, we release the atoms from the ODT in a 15 ms timeof-flight (TOF) expansion after which we record an image of the atoms using a standard low-field absorption imaging technique [12]. Note that we adjust the relative amount of erbium and dysprosium in the final thermal mixture for the specific experiments by independently tuning the MOT loading time for each species between 0.5 and 5 s.

In the isotope combinations and range of magnetic fields that we explore here, we observe two interspecies resonances with widths greater than 1 G (see Table I). We now turn to a more detailed characterization of a feature present in the <sup>168</sup>Er - <sup>164</sup>Dy combination, for which atom loss is shown in Fig. 1(a). We chose to focus on this feature because it is relatively isolated from the many narrow homonuclear and heteronuclear resonances typical of lanthanides. In this experiment, the starting mixture contains  $6.2 \times 10^4$  erbium and  $9.1 \times 10^4$  dysprosium atoms and it is prepared by evaporation at  $B_{ev} = 10.9$  G. In order to compensate for loss during magnetic-field ramps and slow drifts of the atom number, we normalize measurements performed with 200-ms hold times at B<sub>FB</sub> to interleaved measurements at 10-ms hold time at the same field. We further performed independent trap-loss spectra in single-species operation to confirm the interspecies nature of the resonance. Moreover, such scans allow us to identify intraspecies resonances and exclude them from the fit [see empty symbols in Fig. 1(a)]. As shown in the inset for erbium, a high-resolution scan reveals a narrow region with less loss near the center of our broad loss feature, probably due to the influence of a second interspecies resonance-because a resonance contributes a scattering length with a different sign on either side of its pole, the contributions from two nearby resonances may counteract in between them, leading to a reduction in loss. This structure is also visible on the dysprosium loss feature but it is not shown in the inset for ease of reading.

A Gaussian fit to the loss profiles, with known narrow single-species resonance excluded, returns a center value of 13.31(2) and 13.33(4) G and a full width at half maximum (FWHM) value of 1.95(5) and 1.3(1) G for erbium and dysprosium, respectively. The observed difference in the fitted width of the two species can be explained by the imbalance in atom number: because this measurement was performed with fewer erbium atoms than dysprosium, the fractional loss of erbium is higher than that of dysprosium, leading to a



FIG. 1. (a) Trap loss from the 13.5 G resonance in the Bose-Bose mixture <sup>168</sup>Er - <sup>164</sup>Dy (red circles and blue squares points, respectively). Empty symbols correspond to narrow single-species resonances, which we exclude from fits. Each point is an average over four experimental repetitions. For each magnetic field, the atom number recorded after 200 ms of hold time is normalized to that at a short hold time of 10 ms. The lines are the Gaussian fits to the data. The inset shows erbium loss measured in a different dataset with 5-mG resolution, and highlights the structure present on the center of the feature. The same structure is visible also for the dysprosium atoms in the mixture. (b) Interspecies elastic cross section  $\sigma_{ErDy}$ measured across the Feshbach resonance using cross-species thermalization. Each value of  $\sigma_{ErDy}$  is extracted from thermalization data using a numerical model for thermalization that includes temporal variation in atom number and temperature; see main text and the Appendix.

greater saturation of loss and broadening of the erbium loss feature.

### **III. INTERSPECIES THERMALIZATION**

To get insights on its effective strength and width, we perform cross-species thermalization measurements across the resonance [see Fig. 1(b)]. Interspecies thermalization experiments are well established techniques to extract effective thermalization cross sections, which in turn depend on the scattering length [49–51]. While inferring a precise value of the scattering length would require the development of a detailed and rigorous model that accurately captures the temperature dependence of the interspecies and anisotropic dipolar scattering [52], and would go beyond the scope of this work, we are able to determine a thermally averaged scattering cross-section from which we can estimate the width of the resonance.

In this cross-thermalization experiment, we selectively heat dysprosium by means of a near-resonant 421-nm light pulse

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along the vertical direction. We confirmed that the light pulse has no direct measurable effect on erbium. The magnetic field is then jumped to the desired value  $B_{FB}$  and held for a variable amount of time, during which the temperature of erbium rises to equilibrate with dysprosium due to elastic collisions. We record the temperature of the two species along a direction orthogonal to the heating pulse, as the effects of center of mass motion are less prevalent here [53], and use a numerical model to extract a cross section from the rate of thermalization. This simple model assumes an energy independent cross section, an assumption which may break down near resonance where unitarity limits on scattering may become significant.

From these thermalization measurements, we can see a dramatic increase in the scattering cross section near resonance, as one would expect for an interspecies Feshbach resonance. Further, we observe a significant modification of the cross section associated with the resonance over a Gauss-scale range of magnetic fields, similar to the width we observe in loss measurements. For comparison, the expected contribution to the scattering cross section due to dipolar interactions is  $2.4 \times$  $10^{-16}$  m<sup>2</sup>, over an order of magnitude below what we infer near resonance. While the exact relationship between the measured cross section and scattering length is complicated by the finite temperature of our atoms and anisotropic nature of the interactions, an approximate value can be attained through the simple expression for s-wave scattering  $\sigma = 4\pi a_s^2$ , where  $a_s$  is the s-wave scattering length [3]. Our largest measured cross sections, near resonance at 13.5 G, imply a scattering length of roughly 400 a<sub>0</sub>, substantially greater than the dipolar length  $a_D = m\mu_0\mu_{\rm Er}\mu_{\rm Dy}/4\pi\hbar^2 \simeq 139 a_0$  associated with interspecies collisions (a0 is the Bohr radius). In this expression,  $m = m_{\rm Er} m_{\rm Dy} / (m_{\rm Er} + m_{\rm Dy})$  is the reduced mass, and  $\mu_{\rm Er}$ ,  $\mu_{\rm Dy}$ are the ground-state magnetic dipole moments for erbium and dysprosium, respectively.

For an isolated resonance and pure contact interactions, a common way to characterize the resonance width is the parameter  $\Delta$ , given by the difference in magnetic field between the pole of the resonance, at which the thermalization rate is maximal, and the nearest zero crossing in the thermalization rate, which would correspond to a lack of scattering [3]. In lanthanides, the presence of anisotropic dipolar interactions leads to a scattering cross section that does not completely vanish. In addition, multiple narrow and overlapping resonances may be present, which may influence the interpretation of such a width measurement. However, to get a rough estimate of the width of the resonance, we can consider the distance between the resonance pole and the apparent minimum in the thermalization rate at 17 G. This suggests a width of  $\Delta \simeq 3.5$  G.

#### IV. DEPENDENCE OF LOSS ON TEMPERATURE

The dependence of the loss feature on the cloud temperature can provide additional information on the nature of the resonance. For broad resonances, a universal regime is expected to emerge near resonance where the scattering cross section and loss are dictated primarily by the atomic momentum, rather than the scattering length [54]. In this regime, the three-body loss parameter  $L_3$  follows a nearly universal form scaling as  $1/T^2$ , where T is the temperature. Such scaling has

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FIG. 2. Three-body loss coefficient  $L_3$  extracted from onresonance loss measurements at the resonance position for different temperatures (black circles), along with a fit to a  $1/T^2$  scaling (black line), as expected for universal three-body loss. The inset shows the resonance width extracted as FWHM from Gaussian fits to the trap-loss spectra versus cloud temperature for a different dataset. Red circles and blue squares refer to erbium and dysprosium, respectively. The reported temperature comes from a TOF estimation.

been observed in broad resonances of several atomic species [54-56].

We observe a temperature dependence of the loss rate near resonance that is suggestive of such universal behavior. By varying the final depth of the ODT reached during evaporation, we tune the temperature of the atomic mixture. For each temperature, we measure atom loss on resonance at 13.4 G as a function of the hold time. We then use a numerical model to extract the rate of interspecies three-body loss, and  $L_3$ .

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These loss coefficients are plotted as a function of temperature in Fig. 2, along with a fit to a  $1/T^2$  dependence, which provides a reasonable description of our data. The universal temperature dependence arises from a maximum value of  $L_3$ possible at a given temperature, given by

$$L_{3,\max} = \frac{\lambda_{3,\max}}{T^2} \simeq \frac{\hbar^5}{m^3} \frac{36\sqrt{3}\pi^2}{(k_{\rm B}T)^2}.$$
 (1)

Factors associated with Efimov physics can lead to a lower value for  $L_3$ , but not higher [54,57,58]. From our fit to a  $1/T^2$  dependence for our data, we extract a value of  $\lambda_3 = 1.0(2) \times 10^{-24} \mu \text{K}^2 \text{cm}^6 \text{ s}^{-1}$ , which is compatible with the predicted bound of  $\lambda_{3,\text{max}} = 2.4 \times 10^{-24} \mu \text{K}^2 \text{cm}^6 \text{s}^{-1}$ .

A reduction in the peak loss rate with increasing temperature can also result from thermal broadening of the resonance, especially for very narrow resonances [45]. This is unlikely to be the dominant effect here, as for typical differential magnetic moments between entrance and closed channels in our lanthanide system [59], we would expect broadening on the scale of a few times 10 mG for temperatures near 1  $\mu$ K, much narrower than the Gauss-scale width of our feature. Further, suppression of peak loss is typically accompanied by a commensurate broadening and shift of the loss feature on the scale of its width, which we do not observe (inset in Fig. 2).

#### V. SURVEY OF NARROW RESONANCES

In addition to the few relatively broad resonances, the lanthanides exhibit many narrow resonances, whose statistical properties have been investigated for single-species gases [43,45,46]. In this section we compare the abundance of interspecies resonances to single-species resonances by performing high-resolution trap-loss spectroscopy on the isotope combination  $^{166}\text{Er} - ^{164}\text{Dy}$  (see Fig. 3). Here, we investigate four different magnetic-field ranges, each 10 G wide,



FIG. 3. High-resolution trap-loss spectroscopy for a balanced mixture of  $^{166}$ Er and  $^{164}$ Dy (red and blue curves respectively), with initial atom numbers of roughly  $10^5$  per species at a temperature of 500 nK after 400 ms of interaction time. The magenta ticks indicate the heteronuclear resonance positions as extracted by our analysis (see main text). The measurement is composed of four datasets [0, 10] G, [22, 32] G, [50, 60] G, and [60, 70] G with a stepsize of 5 mG. Each point is an average over four experimental repetitions. Atom numbers are normalized to the maximum of each dataset for ease of reading. The broad loss feature in Dy near 68.8 G was not observed in previous work [45], and may result from a technical source of loss in our experiment.



FIG. 4. (a)–(d) Staircase function describing the number of Feshbach resonances as a function of the four investigated magnetic-field ranges: [0, 10] G, [22, 32] G, [50, 60] G, and [60, 70] G respectively. The black line shows the number of heteronuclear resonances. The red and blue lines show the number of homonuclear resonances for <sup>166</sup>Er and <sup>164</sup>Dy, respectively. The shaded areas represent our confidence intervals (see main text).

with a resolution 40 times higher than the one used for the exploratory surveys. To enable direct comparison with the previous works performed on single species [43,45], we use similar experimental conditions (isotope, atom number, temperature, and hold time).

As expected, we observe many narrow homonuclear resonances [43,45]. In addition, we also identify many narrow heteronuclear resonances. To distinguish these two types of resonance, we first label features with a fractional loss above 30% as resonances. We then categorize these resonances as interspecies if erbium and dysprosium loss features occur simultaneously within a range of  $\pm 10$  mG and with a loss amplitude ratio in the range 0.5–2. Features that do not meet both of these criteria, are labeled either as homonuclear or ambiguous, based on comparison with separate scans performed with single species, either within this work or from previously published data [43,45]. The numbers of ambiguous features define our confidence intervals.

In order to visualize the number of resonances, we construct the staircase function  $\mathcal{N}(B)$ , which describes the cumulative number of resonances from the start of a scan range up to a given magnetic field  $B_{\rm FB}$ . Figures 4(a)-4(d) shows  $\mathcal{N}(B)$  for the four investigated magnetic-field ranges. The black lines represent heteronuclear Feshbach resonances, while the red and the blue lines represent the homonuclear <sup>166</sup>Er and <sup>164</sup>Dy resonances, respectively. The shaded regions represent our confidence

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interval defined by the total number of ambiguous Feshbach resonances.

Our analysis results in a total number of heteronuclear resonances of  $\mathcal{N}_{ErDy}(tot) = 339(16)$ , counting all magnetic-field ranges, and a number of homonuclear resonances of  $\mathcal{N}_{Er}(tot) = 116(16)$  and  $\mathcal{N}_{Dy}(tot) = 144(16)$ . Within our confidence intervals, we detect a total number of homonuclear resonances comparable with those of previous works [43,45]. The corresponding total density of resonances  $\bar{\rho}$ , given by the total number of resonances divided by the total range of magnetic fields scanned are  $\bar{\rho}_{ErDy} = 8.5(4) \, \mathrm{G}^{-1}$ ,  $\bar{\rho}_{Er} = 2.9(4) \, \mathrm{G}^{-1}$ , and  $\bar{\rho}_{Dy} = 3.6(4) \, \mathrm{G}^{-1}$ .

For our combined dataset, we find that the total number of heteronuclear resonances exceeds the combined number of homonuclear resonances for the two species:  $\bar{\rho}_{ErDy} = \alpha(\bar{\rho}_{Er} +$  $\bar{\rho}_{\rm Dy}$ ), with  $\alpha = 1.3(2)$ . We would expect that the average density of heteronuclear resonances should be greater than the sum of the two homonuclear resonance densities. This is because each species contributes a set of internal states that can be coupled to, and the heteronuclear resonances are not subject to the same symmetrization requirements as the homonuclear resonances. In resonances involving distinguishable particles, both gerade and ungerade Born-Oppenheimer molecular potentials contribute, as well as both even and odd partial waves for the entrance channel. Our data is consistent with this expectation ( $\alpha > 1$ ). Note that we do observe a lower number of interspecies resonances in the range 50-60 G, perhaps as a result of the nonrandom distribution of resonances as observed in the single-species case [43,45], or to the presence of broad homonuclear erbium resonances that could obscure the observation of interspecies resonances.

#### VI. COINCIDENT LOSS FEATURE IN DIFFERENT ISOTOPES

Finally, we have also searched for broad (Gauss-range) resonances in Bose-Fermi mixtures consisting of fermionic <sup>161</sup>Dy combined with different bosonic isotopes of erbium–<sup>166</sup>Er, <sup>168</sup>Er, and <sup>170</sup>Er, as well as Fermi-Fermi mixtures of <sup>161</sup>Dy and <sup>167</sup>Er. For these combinations, we perform only coarse scans and thus only resolve broad features. In mixtures involving the bosonic isotopes of erbium we observe a correlated loss feature between erbium and dysprosium near 161 G (see Fig. 5). This loss feature is not present at our level of measurement sensitivity with either species alone, or in the mixture with the fermionic <sup>167</sup>Er. Surprisingly, the loss feature is centered at the same magnetic field (to within our resolution of 0.1 G) for all bosonic isotopes of erbium. This is quite unexpected as the magnetic-field value of the resonance position is typically highly sensitive to the reduced mass of the atoms involved [47].

To shed more light on this puzzling feature, we conducted further investigations on the exemplary case of  $^{168}\text{Er} - ^{161}\text{Dy}$ . For such a mixture, we performed loss measurements with erbium numbers ranging from  $1.3 \times 10^4$  to  $3 \times 10^4$  (with the number of dysprosium fixed at  $2.5 \times 10^4$ ). We found in each case that the number of erbium lost was roughly equal to the number of dysprosium lost, and that the number of total atoms lost was roughly proportional to the number of erbium present. We further varied the temperature of the mixture from



FIG. 5. Trap loss spectra for fermionic <sup>161</sup>Dy in combination with bosonic <sup>166</sup>Er, <sup>168</sup>Er, and <sup>170</sup>Er, and fermionic <sup>167</sup>Er (a)–(d), respectively. Red circles represent erbium, blues squares represent dysprosium, and lines are Gaussian fits to the losses. The solid vertical gray lines highlight the peak centers from the fit over dysprosium losses. For the plots with bosonic erbium, the atom number after 100 ms of interaction time is normalized to a short hold time of 5 ms. In the plot with <sup>167</sup>Er, the normalization is performed using the maximum value in the dataset. For all panels, each point is an average over four experimental repetitions.

600 to 2000 nK, and did not find a dramatic dependence of the loss coefficient on temperature (modeling loss as either twobody Er-Dy loss or three-body loss with equal coefficients for Er-Er-Dy and Er-Dy-Dy).

Several physical mechanisms could be consistent with such a feature. One possibility is that the resonance we observe is associated with a bound state of a shallow molecular potential [60]. Mechanisms to create such potentials have been proposed for species with dipolar interactions [61,62]. However, none are obviously applicable to magnetic atoms in the lowest energy entrance channel. Further, given the level of insensitivity to the mass of erbium, we would expect to see additional resonances of a shallow potential in the magnetic-field range over which we survey, which we do not. A second possibility is that the feature we observe is not a true interspecies resonance, but rather an intraspecies resonance in dysprosium whose loss rate is enhanced by the presence of bosonic erbium atoms. A similar effect was reported in a mixture of fermionic lithium and bosonic rubidium atoms [63]. Finally, it is possible that this feature is not a Feshbach resonance at all, but rather the result of spin-changing processes resulting from unintentional radio-frequency tones in the laboratory, or of an interspecies photoassociation resonance. We have ruled out the most likely culprits for the last effect by varying the relative detuning between our horizontal and vertical dipole traps and observing no change in the resonance position. We hope that our presentation of this mysterious feature may spur theoretical exploration of possible physical mechanisms.

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#### VII. CONCLUSIONS

In conclusion, we have reported experimental observation of heteronuclear magnetic Feshbach resonances in several isotope mixtures of erbium and dysprosium. Among the Gauss-broad features identified in our surveys, we have characterized one in the combination <sup>168</sup>Er - <sup>164</sup>Dy by means of cross-species thermalization measurement and temperature dependence analysis. We performed high-resolution trap-loss spectroscopy in the combination  $^{166}\text{Er}$  -  $^{164}\text{Dy}$  to compare the average resonance density of the mixture with respect to the single-species case. In mixtures of fermionic <sup>161</sup>Dy and bosonic erbium atoms, we observed a correlated loss feature which appears to be insensitive on the erbium isotope used but absent in dysprosium alone. Our observations pave the way to realize tunable interactions in quantum degenerate mixtures of dipolar atoms. Knowledge of the range of tunability of these interspecies interactions is a key ingredient for varied opportunities including studies of the miscibility of binary condensates, of vortex lattices, and of dipolar polarons [32-37].

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#### APPENDIX: MEASUREMENT AND ANALYSIS DETAILS

#### 1. Cross-species thermalization

As an exemplary case, we study in more detail the resonance found in the  $^{168}{\rm Er}$  -  $^{164}{\rm Dy}$  Bose-Bose mixture near 13.5 G. To reliably quantify the value of the interspecies cross section, we developed the following scheme for cross-species thermalization measurements [49-51]. To avoid heating of the sample by crossing Feshbach resonances, we evaporate the mixture at Bev close to resonance. Specifically, when measuring on the low(high)-field side of the feature we evaporate at  $B_{\text{evap}} = 10.8 \text{ G}(16.4 \text{ G})$ . Once the sample is prepared as previously described (here we use an unbalanced mixture with twice as much Dy as Er), we compress the trap by linearly increasing the horizontal ODT power by a factor of 5 and the vertical ODT power by two in 500 ms to prevent any plain evaporation. The final trap frequencies in the compressed trap are  $\omega_{x,y,z} = 2\pi \times (409, 26, 391) \text{s}^{-1}$ . Subsequently, we ramp the magnetic field in 5 ms to either 10 or 16 G. Here, a pulse of near-resonant 421-nm light propagating along the magnetic field direction (z) is used to selectively heat dysprosium. We fix the duration of the pulse at 5.5 ms to roughly match the trap



FIG. 6. Sample temperature traces for erbium (filled circles) and dysprosium (hollow squares) after dysprosium is heated. Purple, green, and orange correspond to magnetic fields of 12, 13.5, and 17 G, respectively. Fit lines represent the results of the numerical integration of Eq. (A2), which fits the temperature profile of erbium based on its initial value and the dysprosium temperatures. Different initial and final conditions (see main text).

oscillation period along this direction and set the pulse intensity to give the desired temperature increase of the dysprosium cloud (up to  $4 \mu K$ ). We confirmed that the light pulse has no direct measurable effect on erbium. Finally, with a quench fast compared to the shortest thermalization rate, the magnetic field is set to the desired value  $B_{\rm FB}$  and held for a variable amount of time, during which the temperature of erbium rises to equilibrate with dysprosium due to thermalizing collisions (Fig. 6). We note that in the temperature evolution of the clouds, the initial temperatures are slightly different. This behavior is mainly due to different evaporation conditions on the two sides of the resonance, the different strength in the quench to the final  $B_{\rm FB}$ , and the heating caused by the resonance itself. By comparing the two species' temperature, we ensure that these different conditions are consistent with general offsets on the single measurement thus not affecting the final estimation of the cross-section.

To extract a scattering cross section from our cross-species thermalization data, we use a fit to a numerical model for the thermalization of two species. In principle, a simple exponential fit to the temperature difference between the two species could also be used, but does not account for changes in the atom number or average temperature of the sample that may arise from residual evaporation during the thermalization time. Our numerical model follows that of Ref. [49]. We treat the scattering cross section as independent of the energy of the colliding particles, an assumption that greatly simplifies the analysis, but inevitably breaks down near enough to resonance where unitarity considerations bound the scattering cross section. This assumption leads to a collision rate for each atom PHYSICAL REVIEW A 102, 033330 (2020)

of species 1 with atoms of species 2 given by

r

$$y_{12} = \frac{N_2 m_1^{3/2} \bar{\omega}_1^3}{\pi^2 k_{\rm B} (T_1 + \beta^{-2} T_2)^{3/2}} \sqrt{\frac{T_1}{m_1} + \frac{T_2}{m_2}} \sigma_{12}, \qquad (A1)$$

where  $m_1$ ,  $m_2$ ,  $T_1$ , and  $T_2$  are the masses and temperatures of species 1 and 2,  $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$  characterizes the frequency of the trap,  $\beta^2 = m_2 \bar{\omega}_2^2 / m_1 \bar{\omega}_1^2$ , and  $\sigma_{12}$  is the effective interspecies cross section. We assume that the energy exchanged per collision is given by  $\Delta E = \xi k_{\rm B} (T_2 - T_1)$  where  $\xi = 4m_1 m_2 / (m_1 + m_2)^2$ , and that the heat capacity of each atom is  $3k_{\rm B}$ . This leads to a differential equation for the temperature of erbium:

$$\frac{dT_{\rm Er}}{dt} = \frac{\xi k_{\rm B} (T_{\rm Dy} - T_{\rm Er}) N_{\rm Dy} m_{\rm Er}^{3/2} \bar{\omega}_{\rm Er}^3}{3\pi^2 k_{\rm B} (T_{\rm Er} + \beta^{-2} T_{\rm Dy})^{3/2}} \\ \times \sqrt{\frac{T_{\rm Er}}{m_{\rm Er}} + \frac{T_{\rm Dy}}{m_{\rm Dy}}} \sigma_{\rm ErDy}, \tag{A2}$$

which we can numerically integrate using the instantaneous values for  $T_{\rm Dy}$  and  $N_{\rm Dy}$ , and from this extract the scattering cross section  $\sigma_{\rm ErDy}$  that yields a thermalization profile that best matches our data, as determined through a least-squares difference. Examples of three such fits, for 12, 13.5, and 17 G are shown in Fig. 6, and generally describe our thermalization data well.

#### 2. Temperature dependence of loss

We quantify the temperature dependence of three-body loss in terms of the interspecies three-body loss coefficient. For a single species, the three-body loss coefficient  $L_3$  can be defined by  $\dot{N}/N = -L_3 \langle n^2 \rangle$ , where N is the total number of atoms, and  $\langle n^2 \rangle = \int d^3r \ n^3(r)/N$  represents the average squared density of scattering partners for an atom in the gas. n(r) is the local density of the gas.

We define analogous quantities for our two-species mixture, containing particles denoted i and j. In this case,

$$\frac{\dot{N}_i}{N_i} = \frac{-1}{3N_i} \int d^3r \; (2L_3^{i,i,j} n_i^2(\boldsymbol{r}) n_j(\boldsymbol{r}) + L_3^{j,j,i} n_i(\boldsymbol{r}) n_j^2(\boldsymbol{r})).$$
(A3)

Here,  $L_3^{i,i,j}$  represents the loss rate due to collisions involving two atoms of species *i* and one of *j*.

To arrive at simple expressions, we make several assumptions and approximations. First, we treat the mass, temperature, and polarizability of the two atomic species as equal, which is a reasonable approximation for erbium and dysprosium isotopes in our 1064-nm wavelength ODT [39]. This assumption implies equivalent spatial distributions for the two species, which we assume to be thermal in our threedimensional harmonic trap. We next set  $L_3^{i,i,j} = L_3^{j,j,i} \equiv L_3^i$ near resonance, essentially assuming that the loss process is primarily determined by the two pairwise interactions between the minority participant and the two majority atoms. We find this assumption leads to a model consistent with our observed relative loss between the two species. With these simplifications in place, we define  $L_3^i$  using  $\dot{N}_i/N_i =$  GIANMARIA DURASTANTE et al.

$$-L_3^i \langle n^2 \rangle_{\text{eff}}^i$$
, where

$$\langle n^2 \rangle_{\rm eff}^i = \frac{(2N_iN_j + N_j^2)m^3\bar{\omega}^6}{3^{\frac{5}{2}}8\pi^3(k_{\rm B}T)^3}$$
 (A4)

and  $\bar{\omega} = (\omega_x \omega_y \omega_z)^{1/3}$  is the geometric mean of the trap oscillation frequencies.

We extract the resonant value of  $L_3$  by measuring remaining atom number versus hold time in mixtures prepared at different temperatures, with the magnetic field set near resonance at 13.4 G. We then fit the resulting data by numerically integrating Eq. (A3). Because we observe significant singlespecies loss of erbium (the majority species), we treat the erbium atom number measured at each time step as inputs to our fit, and extract the value of  $L_3$  that best predicts the loss of dysprosium. Here, we assume that  $L_3^{1,i,j} = L_3^{1,j,i} \equiv L_3$ . We bound the effects of single-species loss in dysprosium by repeating the same measurement and analysis protocol off resonance at 11.5 G and 16.5 G. The error bars in Fig. 2 of the main text include a contribution corresponding to the extracted  $L_3$  in the off-resonant condition, which contain both the effects of single-species loss and the small effect

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of off-resonant interspecies loss. Also included are errors associated with the observed change in temperature during the loss measurement, and relating to the approximations made in estimating the density.

In a regime where the scattering length a exceeds the thermal wavelength  $\lambda_{\text{th}} = h/\sqrt{2\pi m k_{\text{B}}T}$ , and thermal broadening is small compared to the width of the loss feature, we expect roughly  $L_3 \propto 1/T^2$ , as has been observed in several experiments involving single atomic species [54-56]. This picture becomes complicated somewhat in the case of a binary mixture due to stronger Efimov effects, which lead to a temperature-dependent modulation of loss relative to the simple  $1/T^2$  prediction. In particular, the parameter  $s_0$ , which characterizes the strength of the three-body Efimov potential, is equal to approximately 1.006 for identical bosons, but approximately 0.41 for our binary mixture [57,58]. The fractional importance of these temperature-dependent modifications scale as  $e^{-\pi s_0}$  [54], making them a minor correction for identical bosons, but a potentially important effect in mixtures. It is possible that such effects contribute to deviations of our data from a  $1/T^2$  form, but a true calculation would require knowledge of short-range inelastic processes in our system.

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## XLII ADDITIONAL PUBLICATIONS

## C.3 TWO-DIMENSIONAL SUPERSOLIDITY IN A DIPOLAR QUANTUM GAS

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## Two-dimensional supersolidity in a dipolar quantum gas

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Supersolidity — a quantum-mechanical phenomenon characterized by the presence of both superfluidity and crystalline order — was initially envisioned in the context of bulk solid helium, as a possible answer to the question of whether a solid could have superfluid properties [1–5]. While supersolidity has not been observed in solid helium (despite much effort)[6], ultracold atomic gases have provided a fundamentally new approach, recently enabling the observation and study of supersolids with dipolar atoms [7–16]. However, unlike the proposed phenomena in helium, these gaseous systems have so far only shown supersolidity along a single direction. By crossing a structural phase transition similar to those occurring in ionic chains [17–20], quantum wires [21, 22], and theoretically in chains of individual dipolar particles [23, 24], we demonstrate the extension of supersolid properties into two dimensions, providing an important step closer to the bulk situation envisioned in helium. This opens the possibility of studying rich excitation properties [25–28], including vortex formation [29–31], as well as ground-state phases with varied geometrical structure [7, 32] in a highly flexible and controllable system.

Ultracold atoms have recently offered a fundamentally new direction for the creation of supersolids — rather than looking for superfluid properties in a solid system like <sup>4</sup>He, ultracold atoms allow one to induce a crystalline structure in a gaseous superfluid, a system which provides far greater opportunity for control and observation. This new perspective has enabled supersolid properties to be observed in systems with spin-orbit coupling [33] or long-range cavity-mediated interactions [34], though in these cases the crystalline structure is externally imposed, yielding an incompressible state. In contrast, dipolar quantum gases of highly magnetic atoms can spontaneously form crystalline structure due to intrinsic interactions [11–13], allowing for a supersolid with both crystalline and superfluid excitations [14–16]. In these demonstrations, supersolid properties have only been observed along a single dimension, as a linear chain of phase-coherent "droplets", i.e. regions of high density connected by low-density bridges of condensed atoms, confined within an elongated optical trap.

The extension of supersolidity into two dimensions is a key step towards creating an ultracold gas supersolid that is closer to the states envisioned in solid helium. Compared to previous studies of incoherent two-dimensional dipolar droplet crystals [8, 35], we work with both a substantially higher atom number N and relatively strong repulsive contact interactions between atoms. This leads to the formation of large numbers of loosely bound droplets, enabling us to establish phase coherence in two dimensions. In our system, the repulsive dipolar interactions between droplets facilitate a structural transition from a linear to a two-dimensional array, analogous to the Coulomb-interaction-mediated structural phase transitions observed with ions [17-20]. Unlike ions however, our droplets are compressible and result from the spontaneous formation of a density wave, allowing for dynamical variation in both droplet number and size. Further, the exchange of particles between droplets enables the spontaneous synchronization of the internal phase of each droplet across the system, and the associated superfluid excitations [14-16].

Dipolar quantum gases exhibit a rich set of groundand excited-state phenomena due to the competition between many energetic contributions. These include mean-field interactions of both contact and dipolar nature, quantum fluctuations, and external confinement, parameterized by potentially anisotropic trapping frequencies  $f_{x,y,z}$ . Such systems can be described with great accuracy by using an extended Gross-Pitaevskii equation (eGPE) [36–39]. Even a fine variation of the



FIG. 1. Calculated phases of dipolar droplet array. a. In-trap ground-state density profiles calculated using eGPE for atom numbers  $N \in [3.3, 4.4, 5.8] \times 10^4$  in the droplets and trap aspect ratios  $\alpha_t = f_x/f_y \in [0.33, 0.35, 0.39]$  (left to right). The scattering length  $a = 88 a_0$ , where  $a_0$  is the Bohr radius. Green dots depict the droplet positions obtained from the variational model, assuming the same N and droplet number  $N_D$  as the eGPE. Stars connect to experimentally observed density profiles in Fig. 2b. **b.** Phase diagram, obtained from our variational model, as a function of N and  $\alpha_t$ for  $f_x = 33$  Hz,  $f_z = 167$  Hz. Linear (two-dimensional) phases with  $N_D$  droplets are labelled as  $1D_{N_D}$  ( $2D_{N_D}$ ).

strength of these energetic contributions can lead to dramatic qualitative changes in the state of the system, for example enabling a transition from a uniform condensate to a supersolid, or in our present case, from a linear supersolid to a two-dimensional one.

Fig.1a shows ground-state density profiles calculated across this transition using the eGPE at zero temperature. These profiles feature arrays of high-density

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droplets, immersed in a low-density coherent "halo" that establishes phase-coherence across the system. As the trap becomes more round, the initially linear chain of droplets acquires greater transverse structure, eventually forming a zig-zag state consisting of two offset linear arrays.

Although the eGPE has remarkable predictive power, full simulations in three dimensions are numerically intensive, making a global survey of the array properties as a function of our experimental parameters difficult. To overcome this limitation, we employ a variational ansatz that captures the key behavior of the system, and allows us to disentangle the competing energetic contributions. In this approach, we describe an array of  $N_D$  droplets by the wavefunction  $\psi(\mathbf{r}) = \sum_{i=1}^{N_D} \psi_j(\mathbf{r})$ , where the *j*-th droplet is assumed to be of the form:  $\psi_j(\mathbf{r}) \propto$  $\sqrt{N_j} \exp\left(-\frac{1}{2} \left(\frac{|\boldsymbol{\rho} - \boldsymbol{\rho}_j|}{\sigma_{\boldsymbol{\rho},j}}\right)^{r_{\boldsymbol{\rho},j}}\right) \exp\left(-\frac{1}{2} \left(\frac{|\boldsymbol{z} - \boldsymbol{z}_j|}{\sigma_{\boldsymbol{z},j}}\right)^{r_{\boldsymbol{z},j}}\right), \quad \text{in-}$ terpolating between a Gaussian and a flat-top profile characteristic of quantum droplets [40]. For a given total number of atoms N and droplet number  $N_D$ , energy minimization provides the atom number  $N_i$  in each droplet, as well as their widths  $\sigma_{\rho(z),j},$  exponents  $r_{\rho(z),j},$ and positions  $\rho_j = (x_j, y_j)$ . Repeating this energy minimization as a function of  $N_D$  gives the optimal number of droplets. This model provides a good qualitative description of the overall phase diagram (Fig. 1b), revealing that the interplay between intra-droplet physics and inter-droplet interaction results in a rich landscape of structural transitions as a function of the atom number and the trap aspect ratio  $\alpha_t = f_x/f_y$ .

Several trends are immediately visible from the phase diagram. Larger N and higher  $\alpha_t$  generally produce states with larger numbers of droplets. Further, as with ions, a large number of droplets favors a 2D configuration, while tighter transverse confinement (small  $\alpha_t$ ) favors 1D [17–20]. A transition from 1D to 2D is thus expected when moving towards larger N or to higher  $\alpha_t$ . In stark contrast to the case of ions, the number of droplets typically increases across the 1D to 2D transition, implying a first-order nature, while only narrow regions in the phase diagram may allow for a 1D-to-2D transition at constant droplet number.

The variational results are in excellent agreement with our eGPE numerics, in terms of predicting the qualitative structure of droplet array patterns, as shown in Fig. 1a. Slight discrepancies exist between the two theories regarding the predicted droplet positions and the location of the 1D-to-2D transition. This is likely because of the presence of the halo in the eGPE simulation (and presumably in the experiment), visible in Fig. 1a, which is not accounted for in the variational model. This halo appears to accumulate at the ends of the trap, pushing the droplets toward the trap center and likely increasing the effective trap aspect ratio experienced by the droplets.

To explore the 1D to 2D transition experimentally, we



FIG. 2. Linear to zig-zag transition in an anisotropic trap. a. We confine and condense dipolar <sup>164</sup>Dy atoms within an anisotropic optical dipole trap (ODT) formed by the intersection of two laser beams. By tuning the aspect ratio of the trap in the x-y plane ( $\alpha_t$ ), perpendicular to an applied magnetic field B, we induce a transition between linear and zig-zag configurations of droplets. b. Single-trial images of the in-trap density profile of atoms at different  $\alpha_t$ , showing structural transition from linear to zig-zag states, as well as an increase in droplet number for higher  $\alpha_t$ . Stars indicate values  $\alpha_t$  and N corresponding to the eGPE calculations of Fig. 1a. c. Atomic aspect ratio  $\alpha_a$  versus trap aspect ratio  $\alpha_t$ .  $\alpha_a$  is the ratio of minor to major axes of a two-dimensional Gaussian fit to the imaged in-trap density profile (inset). For the supersolid droplet array (black markers) we see an abrupt change in  $\alpha_a$  at the critical trap aspect ratio  $\alpha_t^*$ , extracted from the fit (gray line, see methods). The shape of the transition agrees well with eGPE prediction (green diamonds, see methods). For an unmodulated condensate (white markers), no abrupt change is evident. d. Distribution of droplet number versus  $\alpha_t$ , showing a distinct increase in droplet number at the transition of linear to zig-zag configurations.

use a condensate of highly magnetic  $^{164}\mathrm{Dy}$  atoms confined within an anisotropic optical dipole trap with independently tunable trap frequencies  $f_{x,y,z}$ . The trap, shown in Fig.2a, is shaped like a surf-board with the tight axis along gravity and along a uniform magnetic field that orients the atomic dipoles and allows tuning of the contact interaction strength. Typically, we perform evaporation directly into our state of interest at our desired final interaction strength, as demonstrated in Refs.[13, 41]. A combination of in-trap and time-of-flight (TOF) imaging provides us with complementary probes of the density profile of our atomic states, and the phase coherence across the system.

We begin by studying the transition from one to two dimensions by changing the strength of transverse confinement provided by the trap. Our optical setup allows us to tune  $f_y$  from roughly 75 to 120 Hz, while leaving  $f_x$ ,  $f_z$  nearly constant at 33(2), 167(1) Hz, and thus to vary the trap aspect ratio  $\alpha_t$  in the plane perpendicular to the applied magnetic field and our imaging axis. For small  $\alpha_t$ , the atoms are tightly squeezed transversely, and form a linear-chain supersolid (as seen in in-trap images of Fig. 2b). As we increase  $\alpha_t$  above a critical value  $\alpha_t^* = 0.34(2)$ , we observe a structural phase transition to a two-dimensional (2D) state with two side-by-side droplets in the center of the chain. By further increasing  $\alpha_t$ , the 2D structure extends to two offset lines of droplets in a zig-zag configuration. The observed patterns match well with the ground-state predictions from the eGPE calculations when we globally fix the scattering length to  $88a_0$ .

We obtain higher atom numbers in the more oblate traps (higher  $\alpha_t$ ), giving  $N = 6.5(5) \times 10^4$  at  $\alpha_t = 0.44$ and  $N = 2.5(4) \times 10^4$  at  $\alpha_t = 0.28$ . This further facilitates the crossing of the 1D to 2D transition, by favoring states with larger numbers of droplets in the broader traps. In the zig-zag regime, two-dimensional modulation is clearly visible for durations beyond one second. Further, the droplet configuration patterns are fairly repeatable, with clear structure visible in averaged images



FIG. 3. Coherence in linear and zig-zag states. Upper panels show averaged images of experimental TOF interference patterns, along with projections along horizontal and vertical directions of average (solid black lines) and individual images (gray lines). The vertical projection is calculated between the dashed lines. Lower panels show interference patterns calculated for the pictured in-trap droplet configurations (green outlines). a. Linear chain of phase-coherent droplets, showing uniaxial modulation persisting in averaged image (26 trials). b. Zig-zag configuration of phase-coherent droplets, showing modulation along two directions that persists in averaged image (51 trials), and hexagonal structure. The spacing of rows in the simulation was adjusted to approximate the observed aspect ratio of TOF image. The image outlined in blue shows the average momentum distribution calculated from a series of 20 variational calculations converging to slightly different droplet configurations, showing the tendency of such fluctuations to broaden features in the interference pattern while maintaining the underlying structure. C. Zig-zag configuration of phase-incoherent droplets. Modulation remains in single images, as evidenced by the spread of gray traces in projection, but washes out in average (43 trials).

as shown in the inset of Fig. 2c, which is an average of 23 trials taken over roughly two hours.

The transition from 1D to 2D is immediately visible when plotting the atomic aspect ratio  $\alpha_a$  versus  $\alpha_t$ , as shown in Fig. 2c. We find that  $\alpha_a$  undergoes a rapid change at  $\alpha_t^*$ , as the single linear chain develops twodimensional structure. For comparison, we plot  $\alpha_a$  measured for an unmodulated BEC, formed at a different magnetic field, which does not feature the sharp kink present for the supersolid state.

In Fig. 2d, we show the number of droplets present for different  $\alpha_t$ . In the 1D regime, we typically see between five and six droplets. This number abruptly jumps up by approximately one droplet for 2D states near the transition point, and then increases up to an average value of eight droplets as  $\alpha_t$  is further increased. The change in droplet number indicates that the transition that we observe is not of simple structural nature, but is also

accompanied by a reconfiguration of atoms within the droplets, as expected from theory (see Fig. 1).

The measurements of in-trap density presented above inform us about the structural nature of the transition, but not about phase coherence, which is the key distinguishing feature between an incoherent droplet crystal and a supersolid. Previous observations of 2D droplet arrays [35] were performed in traps where the ground state is a single droplet [8], and the observed droplet crystal was likely a metastable state lacking inter-droplet phase coherence. In contrast, we expect from our theoretical calculations that the 2D array is the ground state of our surfboard-shaped trap (for  $\alpha_t > \alpha_t^*$ ), facilitating the formation of a phase-coherent, and therefore supersolid state for our experimental parameters.

We experimentally demonstrate the supersolid nature of our 2D modulated state using a matter-wave interference measurement, as previously used in linear supersolid chains [11–13], (Fig. 3a). In this measurement, an array of uniformly spaced droplets creates an interference pattern with spatial period proportional to the inverse of the in-trap droplet spacing. The relative internal phase of the droplets determines both the contrast and spatial phase of the interference pattern [42]. When averaging over many interference patterns, obtained on separate runs of the experiment, clear periodic modulation persists for phase-coherent droplets, but averages out if the relative droplet phases vary between experimental trials. Thus, the presence of periodic modulation in an average TOF image provides a clear signature of supersolidity in our system, as it indicates both periodic density modulation and phase coherence.

Figure 3a shows an example of such an averaged interference pattern for a linear chain. Uniaxial modulation is clearly present along the direction of the chain, indicating a high degree of phase coherence. For comparison, we also show the expected interference pattern calculated for a linear array of four droplets from free-expansion calculations, showing similar structure.

For conditions where in-trap imaging shows a 2D zigzag structure, the averaged interference pattern exhibits clear hexagonal symmetry (Fig. 3b). This is consistent with our expectation, and is indicative of the triangular structure of the underlying state. To confirm that the observed modulation is not present without phase coherence, we repeat the measurement of Fig. 3b at a magnetic field corresponding to independent droplets, and also compute averaged interference pattern for a zig-zag state with the phases of the individual droplets randomized between simulated trials (Fig. 3c). In both cases, the averaged image does not show clear periodic modulation.

By exploiting the transition between linear and zigzag states, we have accessed a regime where the supersolid properties of periodic density modulation and phase coherence exist along two separate dimensions. Future work will focus on further understanding the spectrum of collective excitations in the full two-dimensional system [26–28, 43], where both the crystalline structure and the exchange of particles between droplets will play an important role. Further investigations may elucidate in more detail the nature of the phase transitions and expected configurations in a wider range of trap aspect ratios, as well as the role that defects play in the 2D system, either as phase-slips in the zig-zag patterns [44, 45], or as vortices trapped between droplets of the array [29–31].

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#### Methods

Experimental apparatus and protocols: Our experimental apparatus has been described in detail in Ref. [46]. Here, we evaporatively prepare up to  $N = 6.5(5) \times 10^4$  condensed <sup>164</sup>Dy atoms in a crossed optical dipole trap formed at the intersection of two beams derived from the same 1064 nm laser, although detuned in frequency to avoid interference. One beam (the static ODT) has an approximately 60 µm waist. The second (the scanning ODT) has an 18 µm waist, whose position can be rapidly scanned horizontally at 250 kHz to create a variably anisotropic time-averaged potential. By tuning the power in each beam, and the scanning range of the scanning ODT, we gain independent control of the trap frequencies in all three directions. The two trapping beams propagate in a plane perpendicular to gravity, and cross at a  $45^{\circ}$  angle, which leads to the rotation of the zig-zag state at high  $\alpha_t$  visible in Fig. 2b.

We apply a uniform magnetic field oriented along gravity and perpendicular to the intersecting dipole traps, with which we can tune the strength of contact interactions between atoms. This allows us to create unmodulated Bose-Einstein condensates, supersolid states, or states consisting of independent droplets at fields of B =23.2 G, 17.92 G, and 17.78 G, respectively.

Details of our imaging setup are provided in Ref. [41]. In-trap and TOF images are performed along the vertical direction (along B and gravity), using standard phasecontrast and absorption techniques, respectively. The resolution of our in-trap images is approximately one micron. We use a 36 ms TOF duration for imaging interference patterns.

Atom number: We extract the condensed atom number N from absorption imaging performed along a horizontal direction in a separate set of experimental trials under otherwise identical experimental conditions. This allows for a larger field of view, and better fitting of thermal atoms. N is determined by subtracting the fitted thermal component from the total absorption signal.

For comparison between experiment and theory, and between the variational and eGPE theory methods, we associate N with the number of atoms in the droplets, and not in the diffuse halo that surrounds the droplets. From simulation of TOF expansion, we find that the halo is repelled at early expansion times, and is likely indistinguishable from the thermal cloud in our TOF measurements. While it is possible that some of the halo is counted in N, we neglect this possibility and assume that N includes only atoms within droplets.

**Scattering length:** The positions of phase boundaries between different droplet configurations are quite sensitive to the scattering length a, which is not known with high precision in our range of magnetic fields. For all theory, we use a value of  $a = 88 a_0$ , where  $a_0$  is the Bohr radius, as this value provides good agreement between experiment and theory for the 1D-to-2D transition point. **Extracting critical aspect ratio:** The critical aspect ratio  $\alpha_t^*$  is extracted from fit to the function  $\alpha_a = \alpha_0$  for  $\alpha_t < \alpha_t^*$ ,  $\alpha_a = \sqrt{\alpha_0^2 + b(\alpha_t - \alpha_t^*)^2}$  for  $\alpha_t > \alpha_t^*$ , where  $\alpha_t^*$ ,  $\alpha_0$  and b are fit parameters. The error bars reported in Fig. 2c represent the standard error on the mean, and are smaller than the markers on most points.

Interference patterns: The predicted interference patterns of Fig. 3 are calculated by assuming free expansion of Gaussian droplets. In reality, the droplets are probably not Gaussian, and interactions during TOF expansion may modify the interference pattern. However, the droplet shape primarily effects the envelope of the interference pattern, which is not our primary interest here, and from eGPE simulations, we expect the effects of interactions to be minor, provided that the droplets be come unbound in a time short compared to the TOF, which we verify by both looking at shorter TOFs and comparing the fringe spacing observed in TOF with that expected from the in-trap droplet spacing. The positions and size of the droplets are tuned to provide illustrative interference patterns.

**Droplet number:** We extract the droplet number from our in-trap images using a peak-finding algorithm applied to smoothed images. The algorithm finds the local maxima above a threshold, which is chosen to be 40% of the overall peak value. Each in-trap density distribution is classified as linear array or 2D zig-zag based on the atomic aspect ratio. Finally, the counts with a given droplet number are normalized by the total number of trials to get the probability shown in Fig. 2d. Fluctuations in the number of atoms in a given trial can push droplets above or below the threshold value, contributing to the spread in extracted droplet number for a given  $\alpha_t$ .
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Die vorliegende Arbeit wurde bisher noch nicht in gleicher oder ähnlicher Form als Dissertation eingereicht.

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Maximilian Sohmen